V.F.S. v2.0 (Open-Gate)
Lyapunov Core Proof Formulae

Core extraction from `lyapunov.html`

Abstract

This document collects the core formulae of the Lyapunov proof for V.F.S. v2.0 (Open-Gate). It is not a separate proof text and contains no graphs. It records the base dynamics, normalized variables, Lyapunov functional, active-domain walls, radial corridor, bounded Open-Gate source, memory bounds, non-Zeno estimate, and asymptotic cleansing statements.

Contents

1 Notation and Core Variables

Symbol	Meaning
V	Voluntas: will coordinate.
F	Factum: action coordinate.
σ	Singular resistance.
λ	Sophia: wisdom coordinate.
Ω_P	Brim / dynamic capacity of Pleroma.
u	Synergic intensity from V and F.
Λ_c	Transformation threshold.
μ	Saturation margin relative to the death-boundary.
q	Normalized Sophia, $q = \lambda/\Omega_P$.
r	Normalized intensity, $r = u/\Omega_P$.
h_1, h_2	Normalized will and action.
h_3	Normalized threshold variable.

Core threshold and intensity:

$$u = \sqrt{VF + \varepsilon}, \qquad \Lambda_c = \frac{\gamma}{\delta}, \qquad \varepsilon \geq 0. \tag{1}$$

Transformation/collapse reading:

$$u > \Lambda_c \quad \text{transformation region,} \qquad u < \Lambda_c \quad \text{collapse region.} \tag{2}$$

2 Base V.F.S. Dynamics

Resistance equation:

$$\dot{\sigma} = (\gamma - \delta u)\tanh(\kappa\sigma) = -\delta(u - \Lambda_c)\tanh(\kappa\sigma). \tag{3}$$

Symmetric will–action dynamics:

$$\dot{V} = \mu(aF + c\lambda) - \rho V, \qquad \dot{F} = \mu(aV + c\lambda) - \rho F, \qquad \mu = 1 - \frac{u}{\Omega_P}. \tag{4}$$

Capacity / Brim equation:

$$\dot{\Omega_P} = \alpha\lambda. \tag{5}$$

Closed Microcosm limit:

$$\dot{\lambda} = -\dot{\sigma} = \delta(u - \Lambda_c)\tanh(\kappa\sigma), \qquad \dot{\sigma} + \dot{\lambda} = 0. \tag{6}$$

Open-Gate Sophia dynamics:

$$\dot{\lambda} = -\dot{\sigma} + I_{\text{gate}}(t) = \delta(u - \Lambda_c)\tanh(\kappa\sigma) + I_{\text{gate}}(t). \tag{7}$$

Open balance:

$$\dot{\sigma} + \dot{\lambda} = I_{\text{gate}}(t) \geq 0, \qquad \sigma(t) + \lambda(t) = \sigma_0 + \lambda_0 + \mathcal{G}_{\text{recepta}}(t). \tag{8}$$

Received grace integral:

$$\mathcal{G}_{\text{recepta}}(t) = \int_0^t I_{\text{gate}}(s)\, ds, \qquad \dot{\mathcal{G}}_{\text{recepta}} = I_{\text{gate}}. \tag{9}$$

3 Open-Gate Source

Positive part of Sophia:

$$\lambda_+ = \frac{1}{m} \ln(1 + e^{m\lambda}), \qquad m > 0. \tag{10}$$

Effective gate:

$$g_{\text{eff}}(t) = e^{-\tau t} + \left(1 - e^{-\tau t}\right) \frac{H_K}{H_s + H_K}. \tag{11}$$

Live Open-Gate source:

$$I_{\text{gate}}(t) = \zeta_0\, g_{\text{eff}}(t) e^{-\phi\sigma(t)} \frac{1}{1 + \chi\lambda_+(t)}. \tag{12}$$

Boundedness of the gate:

$$H_K \geq 0, \quad H_s > 0 \quad \Longrightarrow \quad 0 \leq \frac{H_K}{H_s + H_K} < 1, \qquad 0 \leq g_{\text{eff}}(t) \leq 1. \tag{13}$$

Boundedness of live inflow:

$$0 \leq I_{\text{gate}}(t) \leq \zeta_0. \tag{14}$$

4 Resistance Potential

Resistance potential:

$$\Psi_\sigma(\sigma) = \int_0^\sigma \tanh(\kappa s)\, ds = \frac{1}{\kappa} \ln\cosh(\kappa\sigma) \geq 0. \tag{15}$$

Derivative of the resistance potential:

$$\dot{\Psi}_\sigma = \Psi'_\sigma(\sigma)\dot{\sigma} = \tanh(\kappa\sigma)(\gamma - \delta u)\tanh(\kappa\sigma) = (\gamma - \delta u)\tanh^2(\kappa\sigma). \tag{16}$$

Active transformation margin:

$$u \geq \Lambda_c + \eta, \qquad \eta > 0. \tag{17}$$

Dissipation in the active margin:

$$\dot{\Psi}_\sigma \leq -\delta\eta \tanh^2(\kappa\sigma). \tag{18}$$

On $0 \leq \sigma \leq C_\sigma$, there exists $c_\sigma > 0$ such that:

$$\dot{\Psi}_\sigma \leq -c_\sigma \Psi_\sigma. \tag{19}$$

Hence:

$$\Psi_\sigma(t) \le \Psi_\sigma(0)e^{-c_\sigma t}. \tag{20}$$

Near $\sigma = 0$:

$$\Psi_\sigma(\sigma) \sim \frac{\kappa}{2}\sigma^2, \qquad \sigma(t) = O(e^{-c_\sigma t/2}). \tag{21}$$

5 Absolute Barrier Functional

The absolute barrier is a motivational, non-primary barrier guarding absolute boundary faces:

$$\begin{aligned} \mathcal{L}_{abs} = A_\sigma\Psi_\sigma + E_V V^2 + E_F F^2 + E_\lambda \lambda^2 + E_\Delta \frac{(V-F)^2}{2} \\ - \eta_V \ln V - \eta_F \ln F - \eta_\Omega \ln(\Omega_P - \Lambda_c) - \eta_m \ln(\Omega_P - u). \end{aligned} \tag{22}$$

Guarded absolute faces:

$$V = 0, \qquad F = 0, \qquad \Omega_P = \Lambda_c, \qquad u = \Omega_P. \tag{23}$$

6 Normalized Variables

Normalized coordinates:

$$h_1 = \frac{V}{\Omega_P}, \qquad h_2 = \frac{F}{\Omega_P}, \qquad q = \frac{\lambda}{\Omega_P}, \qquad r = \frac{u}{\Omega_P}, \qquad \mu = 1 - r, \qquad h_3 = 1 - \frac{\Lambda_c}{\Omega_P}. \tag{24}$$

Product relation:

$$r^2 = h_1 h_2 + e(h_3), \qquad e(h_3) = \varepsilon \frac{(1-h_3)^2}{\Lambda_c^2}. \tag{25}$$

Closed normalized dynamics:

$$\dot{h}_1 = \mu(a h_2 + c q) - \rho h_1 - \alpha q h_1, \tag{26}$$

$$\dot{h}_2 = \mu(a h_1 + c q) - \rho h_2 - \alpha q h_2, \tag{27}$$

$$\dot{q} = \delta(r + h_3 - 1)\tanh(\kappa\sigma) - \alpha q^2, \tag{28}$$

$$\dot{h}_3 = \alpha q(1 - h_3). \tag{29}$$

Open-Gate normalized q-source:

$$S_q = \frac{I_{\text{gate}}}{\Omega_P} = \frac{\zeta_0 g_{\text{eff}} e^{-\phi\sigma}}{(1 + \chi\lambda_+)\Omega_P}. \tag{30}$$

Open-Gate normalized q-equation:

$$\dot{q} = \delta(r + h_3 - 1)\tanh(\kappa\sigma) - \alpha q^2 + S_q. \tag{31}$$

Bound on S_q inside the active domain:

$$0 \le S_q \le \frac{\zeta_0}{\Omega_P} \le \frac{\zeta_0(1 - h_{3,*})}{\Lambda_c} =: S_q^{\max}. \tag{32}$$

7 Product-Balance Mechanism

Balance defect:

$$D_\Delta = \frac{1}{2}(h_1 - h_2)^2. \tag{33}$$

Let:

$$d = h_1 - h_2. \tag{34}$$

Difference equation:

$$\dot{d} = -(a\mu + \rho + \alpha q)d. \tag{35}$$

Dissipation of balance defect:

$$\dot{D}_\Delta = -2(a\mu + \rho + \alpha q)D_\Delta. \tag{36}$$

If $\mu \geq \mu_* > 0$ and $a > 0$:

$$\dot{D}_\Delta \leq -2a\mu_* D_\Delta. \tag{37}$$

Exponential will–action alignment:

$$D_\Delta(t) \leq D_\Delta(0)e^{-2a\mu_* t}, \qquad |h_1(t) - h_2(t)| \leq |h_1(0) - h_2(0)|e^{-a\mu_* t}. \tag{38}$$

Product floor from radial and threshold walls:

$$\Omega_P \geq \Omega_{\min} := \frac{\Lambda_c}{1 - h_{3,*}}. \tag{39}$$

If $r \geq r_*$ and $h_3 \geq h_{3,*}$:

$$h_1 h_2 = r^2 - \frac{\varepsilon}{\Omega_P^2} \geq p_* := r_*^2 - \frac{\varepsilon}{\Omega_{\min}^2}. \tag{40}$$

Let:

$$d_* := \sqrt{2D^*}. \tag{41}$$

Product-balance lower bound:

$$h_1, h_2 \geq m_{pb} := \frac{\sqrt{d_*^2 + 4p_*} - d_*}{2} > 0. \tag{42}$$

8 Lyapunov Functional

Product-balance Lyapunov core:

$$\mathcal{L}_{pb} = A_\sigma \Psi_\sigma(\sigma) + A_\Delta D_\Delta + A_q q^2. \tag{43}$$

Radial logarithmic barrier:

$$B(r) = -\ln(1 - r), \qquad 0 < r < 1, \qquad r = \frac{u}{\Omega_P}. \tag{44}$$

Non-normalized radial barrier:

$$B(u, \Omega_P) = -\ln\left(1 - \frac{u}{\Omega_P}\right) = \ln\left(\frac{\Omega_P}{\Omega_P - u}\right). \tag{45}$$

Primary V.F.S. Lyapunov functional:

$$\boxed{\mathcal{L}_{\mathrm{VFS}} = \mathcal{L}_{pb} + A_r B(r) = A_\sigma \Psi_\sigma(\sigma) + A_\Delta D_\Delta + A_q q^2 + A_r B(r), \qquad A_\sigma, A_\Delta, A_q, A_r > 0.} \tag{46}$$

Expanded form:

$$\mathcal{L}_{\mathrm{VFS}} = A_\sigma \Psi_\sigma(\sigma) + A_\Delta D_\Delta + A_q q^2 - A_r \ln(1 - r). \tag{47}$$

Non-normalized expanded form:

$$\mathcal{L}_{\mathrm{VFS}} = A_\sigma \Psi_\sigma(\sigma) + A_\Delta D_\Delta + A_q q^2 - A_r \ln\left(1 - \frac{u}{\Omega_P}\right). \tag{48}$$

Radial boundary protection:

$$B(r) \to +\infty \quad \text{as} \quad r \to 1^-. \tag{49}$$

9 Active Domain

Active domain:

$$\mathcal{D}_A = \left\{ \begin{array}{l} 0 \le \sigma \le C_\sigma, \quad q \in [0, q^*], \quad h_3 \in [h_{3,*}, 1), \\ r \in [r_*, r^*], \quad \mu = 1 - r \ge \mu_* > 0, \\ D_\Delta \le D^*, \quad h_1 > 0, \quad h_2 > 0 \end{array} \right\}, \qquad 0 < r_* < r^* < 1. \tag{50}$$

Equivalent death-boundary avoidance inside $\mathcal{D}_A$:

$$0 < r < 1 \qquad \Longleftrightarrow \qquad 0 < u < \Omega_P. \tag{51}$$

Closed upper q-wall:

$$\alpha(q^*)^2 \ge \delta r^*. \tag{52}$$

Open-Gate upper q-wall:

$$\boxed{\alpha(q^*)^2 \ge \delta r^* + S_q^{\max}.} \tag{53}$$

Lower q-wall in Open-Gate:

$$q = 0 : \qquad \dot{q} = \delta(r + h_3 - 1)\tanh(\kappa\sigma) + S_q \ge 0 \tag{54}$$

provided the active margin makes $r + h_3 - 1 \ge 0$.

10 Radial Corridor

Radial identity from $r^2 = h_1 h_2 + e(h_3)$:

$$2r\dot{r} = \mu\big[a(h_1^2 + h_2^2) + cq(h_1 + h_2)\big] - 2\rho(r^2 - e(h_3)) - 2\alpha q r^2 =: \mathcal{R}. \tag{55}$$

Maximum e on the active corridor:

$$e_*^{\max} = \varepsilon \frac{(1 - h_{3,*})^2}{\Lambda_c^2}. \tag{56}$$

Non-degeneracy:

$$r_*^2 > e_*^{\max}. \tag{57}$$

Active margin:

$$r_* + h_{3,*} - 1 \geq \eta \frac{1 - h_{3,*}}{\Lambda_c}. \tag{58}$$

Lower radial wall:

$$a(1 - r_*)(r_*^2 - e_*^{\max}) \geq (\rho + \alpha q^*) r_*^2. \tag{59}$$

Upper radial wall:

$$(1 - r^*)\big[a(d_*^2 + 2r^{*2}) + cq^* \sqrt{d_*^2 + 4r^{*2}}\big] \leq 2\rho(r^{*2} - e_*^{\max}). \tag{60}$$

Direct upper radial inwardness condition:

$$\mu \left[a(h_1^2 + h_2^2) + cq(h_1 + h_2)\right] \leq 2\rho(r^{*2} - e) + 2\alpha q r^{*2} \qquad \text{on } r = r^*. \tag{61}$$

Open-Gate radial convention:

$$q_{\text{new}}^* := \sqrt{\frac{\delta r^* + S_q^{\max}}{\alpha}}. \tag{62}$$

Conservative lower radial wall with q_{new}^*:

$$a(1 - r_*)(r_*^2 - e_*^{\max}) \geq (\rho + \alpha q_{\text{new}}^*) r_*^2. \tag{63}$$

Epektasis-dominance derivative at the upper radial wall:

$$\left.\frac{\partial \mathcal{R}}{\partial q}\right|_{r^*} = \mu c(h_1 + h_2) - 2\alpha r^{*2}. \tag{64}$$

Worst-case bound:

$$\left.\frac{\partial \mathcal{R}}{\partial q}\right|_{r^*} \leq (1 - r^*) c \sqrt{d_*^2 + 4r^{*2}} - 2\alpha r^{*2}. \tag{65}$$

Epektasis-dominance sufficient condition:

$$\boxed{2\alpha r^{*2} \geq (1 - r^*) c \sqrt{d_*^2 + 4r^{*2}} \quad \Longrightarrow \quad \frac{\partial \mathcal{R}}{\partial q} \leq 0.} \tag{66}$$

11 Lyapunov Dissipative Estimate

Derivative decomposition:

$$\dot{\mathcal{L}_{\text{VFS}}} = A_\sigma \tanh(\kappa\sigma)\dot{\sigma} + A_\Delta \dot{D}_\Delta + 2A_q q\dot{q} + A_r \dot{B}(r). \tag{67}$$

Open-Gate contribution to the q^2 block:

$$\frac{d}{dt}(A_q q^2) = 2A_q q \left[\delta(r + h_3 - 1)\tanh(\kappa\sigma) - \alpha q^2\right] + 2A_q q S_q. \tag{68}$$

Bound on Open-Gate contribution:

$$0 \le 2A_q q S_q \le 2A_q q^* S_q^{\max}. \tag{69}$$

Dissipative Lyapunov estimate:

$$\boxed{\dot{\mathcal{L}}_{\mathrm{VFS}} \le C - C_1 \mathcal{L}_{\mathrm{VFS}}, \qquad C_1 > 0, \qquad C < \infty.} \tag{70}$$

Open-Gate constant split:

$$C = C_{\mathrm{closed}} + \Delta C_{\mathrm{gate}}, \qquad 0 \le \Delta C_{\mathrm{gate}} \le 2A_q q^* S_q^{\max}. \tag{71}$$

Key structural point: Here C_1 is unchanged by the Open-Gate source, while C receives only a bounded gate contribution.

12 Main Theorem Formulae

Positive invariance:

$$x(0) \in \mathcal{D}_A \quad \Longrightarrow \quad x(t) \in \mathcal{D}_A \quad \forall t \ge 0. \tag{72}$$

Reset admissibility:

$$x_j^+ \in \mathcal{D}_A, \qquad \mathcal{L}_{\mathrm{VFS}}(x_j^+) \le L_R. \tag{73}$$

Bound from the Lyapunov inequality:

$$\mathcal{L}_{\mathrm{VFS}}(t) \le L_* := \max\left\{L_R, \frac{C}{C_1}\right\}. \tag{74}$$

Radial barrier implication:

$$A_r B(r) \le L_*. \tag{75}$$

Explicit radial bound:

$$-\ln(1-r) \le \frac{L_*}{A_r} \quad \Longrightarrow \quad r(t) \le 1 - e^{-L_*/A_r} < 1. \tag{76}$$

Death-boundary avoidance:

$$r(t) < 1 \quad \Longrightarrow \quad u(t) < \Omega_P(t). \tag{77}$$

Product-balance positivity:

$$h_1, h_2 \ge m_{pb} > 0. \tag{78}$$

No artificial q-floor:

$$q \ge 0 \quad \text{suffices; no assumption } q \ge q_0 > 0 \text{ is required.} \tag{79}$$

13 Non-Zeno Estimate

Hybrid margin:

$$\bar{h}_* = \min\left\{m_{pb}, h_{3,*}, r_*, 1 - r^*, q^*, C_\sigma, \sqrt{D^*}\right\} > 0. \tag{80}$$

Bounded vector field:

$$\bar{M}_* < \infty. \tag{81}$$

Dwell-time lower bound:

$$\Delta t_j \geq \Delta t_* := \frac{\bar{h}_*}{\bar{M}_*} > 0. \tag{82}$$

Non-Zeno conclusion:

$$\sum_j \Delta t_j = +\infty. \tag{83}$$

14 Memory and Gate Bounds

Mensura kernel:

$$R(z) = \frac{z}{z+K}, \qquad z \geq 0, \qquad 0 \leq R(z) \leq 1. \tag{84}$$

Saturated gate memory:

$$\frac{H_K}{H_s + H_K} \in [0, 1) \qquad \forall H_K \geq 0. \tag{85}$$

Gate saturation:

$$g_{\text{eff}} = e^{-\tau t} + (1 - e^{-\tau t})\frac{H_K}{H_s + H_K} \in [0, 1]. \tag{86}$$

Memory velocity bounds:

$$|\dot{H}_K| \leq \eta\,[R((\dot{x}_0)_+) + R((\dot{x}_0)_-)] \leq 2\eta. \tag{87}$$

Wound-memory velocity bound:

$$|\dot{W}| \leq 2\eta. \tag{88}$$

Bounded-gain loop:

$$H_K \xrightarrow{\leq 1} g_{\text{eff}} \xrightarrow{\leq S_q^{\max}} S_q \to \lambda \to V, F \to u \to \dot{x}_0 \xrightarrow{R \leq 1} \dot{H}_K. \tag{89}$$

Extended field bound:

$$\bar{M}_* < \infty \quad \text{including} \quad |\dot{H}_K|, |\dot{W}| \leq 2\eta. \tag{90}$$

15 Asymptotic Cleansing and Alignment

Will–action alignment:

$$D_\Delta(t) \leq D_\Delta(0)e^{-2a\mu_* t}, \qquad |h_1(t) - h_2(t)| \leq |h_1(0) - h_2(0)|e^{-a\mu_* t} \to 0. \tag{91}$$

Resistance cleansing under persistent active margin:

$$u(t) \geq \Lambda_c + \eta \quad \forall t \geq 0 \quad \Longrightarrow \quad \Psi_\sigma(t) \leq \Psi_\sigma(0) e^{-c_\sigma t} \quad \Longrightarrow \quad \sigma(t) \to 0. \tag{92}$$

Open-balance asymptotic Sophia:

$$\lambda(t) = \sigma_0 + \lambda_0 + \mathcal{G}_{\text{recepta}}(t) - \sigma(t). \tag{93}$$

If $\mathcal{G}_{\text{recepta}}(\infty) < \infty$ and $\sigma(t) \to 0$:

$$\lambda(t) \to \sigma_0 + \lambda_0 + \mathcal{G}_{\text{recepta}}(\infty). \tag{94}$$

If $\liminf_{t\to\infty} \lambda(t) > 0$:

$$\Omega_P(t) \to +\infty. \tag{95}$$

Epektatic dilution of S_q:

$$\int_0^\infty \lambda(t)\, dt = +\infty \quad \Longrightarrow \quad \Omega_P(t) \to +\infty \quad \Longrightarrow \quad S_q(t) \to 0. \tag{96}$$

Proof of the last implication:

$$0 \leq S_q(t) \leq \frac{\zeta_0}{\Omega_P(t)} \to 0. \tag{97}$$

16 Numerical Witness Formulae

One admissible parameter point recorded in the Lyapunov file:

$$\begin{gathered} \alpha = 0.366, \quad \delta = 0.610, \quad \rho = 0.237, \quad a = 2.352, \quad c = 0.342, \\ \zeta_0 = 0.507, \quad h_{3,*} = 0.412, \quad r_- = 0.453, \quad r_+ = 0.955, \\ e_{\max} = 0.055, \quad d = 0.107, \quad S_q^{\max} = 0.298, \quad q_{\text{new}} = 1.551. \end{gathered} \tag{98}$$

Upper q-wall witness:

$$\alpha q_{\text{new}}^2 \geq \delta r_+ + S_q^{\max}. \tag{99}$$

Epektasis-dominance witness:

$$2\alpha r_+^2 \geq (1 - r_+) c \sqrt{d^2 + 4r_+^2}. \tag{100}$$

17 Final Proof Chain

Core proof chain:

$$\boxed{\text{Admissibility} \Longrightarrow \mathcal{D}_A \Longrightarrow \dot{\mathcal{L}}_{\text{VFS}} \leq C - C_1 \mathcal{L}_{\text{VFS}} \Longrightarrow r(t) < 1 \Longrightarrow \text{NonZeno.}} \tag{101}$$

Active transformation conclusion:

$$u \geq \Lambda_c + \eta \quad \Longrightarrow \quad \sigma(t) \to 0, \qquad h_1(t) - h_2(t) \to 0. \tag{102}$$

The theorem is an active-domain theorem, not global stability on all phase space: The active-domain assumption is essential:

$$x(0) \in \mathcal{D}_A. \tag{103}$$

Closed limit:

$$\zeta_0 = 0 \quad \Longrightarrow \quad S_q = 0, \quad I_{\text{gate}} = 0, \quad \dot{\lambda} = -\dot{\sigma}, \quad \sigma + \lambda = \sigma_0 + \lambda_0. \tag{104}$$

Open-Gate theorem summary: Open-Gate adds only bounded S_q in $\dot{q}$; it strengthens walls but preserves the dissipative core.