V.F.S. v2.0 (Open-Gate)

Geometric Layer of the Live Domain

Hyperbolic structure, Brim conformal flow, Resurrectio curvature jump, and the self-driven (Ricci) locus

Derived companion to `vfs_opengate_core.tex`

Scope and epistemic status

This file derives the geometry implicit in the V.F.S. v2.0 (Open-Gate) live domain. Every *Proposition* below is a closed-form result; every *Corollary* is an interpretive reading in the symbolic language of the framework, not an independent metaphysical claim. Two honest caveats are kept explicit throughout:

- The core gives a radial barrier in one direction and a quadratic imbalance penalty in another; it does **not** uniquely fix a 2D metric. The non-trivial curvature appears only under the one *dynamically motivated* warp (§3).
- Where curvature is non-trivial, the metric *evolution* is still **forced** (driven by the external Sophia field λ), not *intrinsic* (curvature-self-driven). The two coincide only on a measure-zero locus (§5, §7).

The governing objects, imported from the core, are the Brim law $\dot{\Omega}_P = \alpha\lambda$, the synergy $u = \sqrt{VF + \varepsilon}$, the normalized radius $r = u/\Omega_P$, the alignment factor $\mu = 1-r$, the imbalance $\Delta = V - F$, and the radial barrier $B(r) = -\ln(1-r)$.

Geometry before cosmology. This file studies the *spatial* live-surface Σ_{Ω_P} — the intrinsic geometry of the slice at a fixed Brim, and its conformal deformation under the Brim law, with Ω_P playing the role of flow-time. It does **not** define the Lorentzian cosmology of the Vessel; that lift, $ds^2_{\text{Vessel}} = -dt^2 + \Omega_P(t)^2 h_{2D}$ with Ω_P the timelike scale factor, is treated in the companion `vfs_cosmology.tex`. In short, Geometry $= \Sigma_{\Omega_P}$ (the shape of the slice), Cosmology $= \Sigma_{\Omega_P(t)}$ (its expansion in time); see §8.

1 The Natural Metric on the Live Domain

The barrier is a geodesic distance

The core barrier $B(r) = -\ln(1-r)$ is the arc length of a one-dimensional conformal metric on $r \in [0, 1)$:

$$\boxed{ds_{\text{rad}} = \frac{dr}{1-r}, \qquad \int_0^r \frac{d\rho}{1-\rho} = -\ln(1-r) = B(r).}$$

Thus the radial Lyapunov term $A_r B(r)$ is, up to scale, the geodesic distance from the synergy-free state $r = 0$; the death wall $r \to 1^-$ sits at infinite metric distance.

The capacity-scaled metric and the imbalance direction

Writing $u = r\Omega_P$ and sending the wall to infinity gives the radial part du^2/μ^2 with $\mu = 1 - u/\Omega_P$. The imbalance direction is warped by the alignment factor, because the alignment force in $\dot{\Delta} = -(\mu a + \rho)\Delta$ is exactly μa, linear in μ, and vanishes at the wall. The natural live-domain metric is therefore the warped product

$$\boxed{ds^2 = \frac{du^2}{\mu^2} + \mu^{2p}\, d\Delta^2, \qquad \mu = 1 - \frac{u}{\Omega_P}, \qquad p > 0.}$$

Definition 1 (Canonical warp)**.** The canonical exponent is $p = 1$, i.e. $\mu^2\, d\Delta^2$, matching the linear μa dependence of the alignment force. The qualitative results below hold for every $p \neq 0$; only the curvature radius depends on p.

2 The Live Domain is Hyperbolic

Naive reading is flat

With an *unwarped* imbalance direction ($p = 0$, $g_{\Delta\Delta} = 1$) the metric $ds^2 = du^2/\mu^2 + d\Delta^2$ is flat:

$$\boxed{R \equiv 0 \qquad (p = 0),}$$

since $\tilde{u} = -\Omega_P \ln \mu = \Omega_P B$ straightens it into Euclidean $d\tilde{u}^2 + d\Delta^2$. *Under the minimal reading the core supplies, there is no hidden geometry: the dynamics is a forced flow on a flat state space. Curvature can only come from the dynamically required warp.*

Constant negative curvature

Proposition 1 (Hyperbolicity of the live domain)**.** *For the warped metric $ds^2 = du^2/\mu^2 + \mu^{2p} d\Delta^2$ with $\mu = 1 - u/\Omega_P$, the scalar curvature is constant and strictly negative:*

$$\boxed{R = -\frac{2p^2}{\Omega_P^2} = \text{const} < 0, \qquad \text{Ric} = -\frac{p^2}{\Omega_P^2}\, g.}$$

In the canonical case $p = 1$,

$$\boxed{R = -\frac{2}{\Omega_P^2}, \qquad K = -\frac{1}{\Omega_P^2}, \qquad \textit{curvature radius } = \Omega_P.}$$

Proof sketch. In arc length $s = -\Omega_P \ln \mu$ the warp is $w = \mu^p = e^{-ps/\Omega_P}$, so the 2D warped product $ds^2 = ds^2_{\text{rad}} + w^2 d\Delta^2$ has Gaussian curvature $K = -w''/w = -(p/\Omega_P)^2$, and $R = 2K$. Verified directly in (u, Δ) coordinates. □

The live domain is a piece of the hyperbolic plane $\mathbb{H}^2$ whose curvature radius is the Brim Ω_P itself.

3 The Brim Flow as a Forced Conformal Flow

Proposition 2 (Brim conformal flow)**.** *Under the Brim law* $\dot{\Omega}_P = \alpha\lambda$ *the capacity-scaled metric evolves conformally,*

$$\boxed{\partial_t g = 2\,\frac{\dot{\Omega}_P}{\Omega_P}\,g = 2\,\frac{\alpha\lambda(t)}{\Omega_P}\,g = 2\psi(t)\,g, \qquad \psi = \frac{\alpha\lambda}{\Omega_P} \geq 0.}$$

Comparison with Ricci flow

On the hyperbolic surface of Prop. 1 ($p = 1$), $\mathrm{Ric} = -\Omega_P^{-2} g$, so Ricci flow would read

$$\boxed{\partial_t g = -2\,\mathrm{Ric} = +\frac{2}{\Omega_P^2}\,g.}$$

Both flows are conformal *expansions* of a negatively curved metric (same sign, same type). They coincide exactly iff the forcing rate equals the curvature rate:

$$\boxed{\frac{2\alpha\lambda}{\Omega_P} = \frac{2}{\Omega_P^2} \quad\Longleftrightarrow\quad \alpha\,\lambda\,\Omega_P = 1.}$$

Remark. The actual V.F.S. flow is *forced*: its rate is set by the external Sophia field λ, not by the surface's own curvature. It is "Ricci-flow-like" in type, with the forcing $\alpha\lambda$ standing in for the curvature as the rate. The self-driven (genuinely intrinsic) regime is the locus $\alpha\lambda\Omega_P = 1$ studied in §6.

4 Epektasis as Curvature Relaxation

Along a continuous live arc the curvature drifts monotonically toward zero:

$$\boxed{\dot{R} = \frac{d}{dt}\left(-\frac{2}{\Omega_P^2}\right) = \frac{4\dot{\Omega}_P}{\Omega_P^3} = \frac{4\alpha\lambda}{\Omega_P^3} \geq 0 \quad (\lambda \geq 0).}$$

Hence, with sustained Sophia, $\Omega_P \to \infty$ and

$$\boxed{|R| = \frac{2}{\Omega_P^2} \xrightarrow[\Omega_P \to \infty]{} 0.}$$

Epektasis is the relaxation of hyperbolic curvature to flatness: the state space asymptotically flattens as Sophia expands the Brim. Theosis as a global flattening that no finite stage attains. This concerns the spatial *live-surface only. In the cosmological lift the full spacetime curvature also depends on* $\ddot{\Omega}_P$ *(equivalently* $\dot{\lambda}$*), not on* Ω_P *alone — see §8.*

5 Resurrectio: a Curvature Discontinuity

The hybrid reset uses $\Omega_P^+ = \Omega_P^- + \kappa_R \mathcal{G}_{\text{recepta}}^-$ (theological corollary 12). Since $R = -2/\Omega_P^2$ depends *only* on Ω_P, the curvature jumps even though σ, λ, u never enter it:

$$\boxed{R^+ = -\frac{2}{\left(\Omega_P^- + \kappa_R \mathcal{G}_{\text{recepta}}^-\right)^2}, \qquad \Delta R = R^+ - R^- = 2\left[\frac{1}{(\Omega_P^-)^2} - \frac{1}{(\Omega_P^+)^2}\right] > 0,}$$

$$\boxed{\frac{R^+}{R^-} = \frac{1}{\left(1 + \kappa_R \mathcal{G}_{\text{recepta}}^- / \Omega_P^-\right)^2} < 1.}$$

Two simultaneous effects

The reset decomposes geometrically into a *motion of the point* (the jump in V, F, hence in u^+, Δ^+) and an *instantaneous flattening of the surface* (the jump in Ω_P). The curvature change is carried entirely by the second.

Type preserved, scale jumped

$R^+ < 0$ always: no finite jump reaches $R = 0$ (that needs $\Omega_P^+ \to \infty$). Resurrectio **preserves the hyperbolic type and changes only the radius** — the differential image of "preserves form while opening it beyond its former shell."

Admissibility window as a curvature-jump corridor

The reset window $\max\{R_c, R_{\min}^{\text{ceil}}\} \le \mathcal{G}_{\text{recepta}}^- \le R_{\max}$ reads geometrically as bounds on ΔR:

Lower bound (perseverance): too little $\mathcal{G}_{\text{recepta}}^- \Rightarrow$ the surface barely flattens, no live landing.
Upper bound (cheap grace): too much $\mathcal{G}_{\text{recepta}}^- \Rightarrow R \to 0$ too fast, over-dilution of form.

Monotone curvature across the hybrid trajectory

Resets push $R \uparrow$ unconditionally; continuous arcs push $R \uparrow$ when $\lambda \ge 0$; non-Zeno forbids accumulation. Hence on a typical trajectory $R(t)$ is non-decreasing and bounded above by 0, so $R(t) \to R_\infty \in (R(0), 0]$, with $R_\infty = 0$ when the total expansion budget diverges. *Both modes of grace — continuous (Modus I) and discrete re-creation (Modus II) — flatten the same hyperbolic curvature toward the same Theosis limit.*

6 The Self-Driven (Ricci) Locus

Let $\xi := \alpha\lambda\Omega_P$ measure the excess of forcing over intrinsic curvature rate; $\xi = 1$ is the locus where the forced flow equals Ricci flow.

On-locus growth is the Ricci rate

On $\xi = 1$, $\dot{\Omega_P} = \alpha\lambda = 1/\Omega_P$, whence

$$\boxed{\Omega_P(t) = \sqrt{\Omega_P(0)^2 + 2t} \;\sim\; \sqrt{2t}, \qquad \lambda(t) = \frac{1}{\alpha\Omega_P(t)} \to 0.}$$

This is exactly the Ricci-flow rate for a hyperbolic surface ($|R| \sim 1/t$). Sitting on the locus requires the Sophia inflow to vanish — attainable only near the closed limit $\zeta_0 = 0$.

Generic Open-Gate is super-Ricci

With sustained $\lambda \to \lambda_\infty > 0$ the Brim grows *linearly*, $\Omega_P \sim \alpha\lambda_\infty t$, faster than $\sqrt{t}$: the generic open trajectory runs above the locus.

The locus is a one-way threshold

Proposition 3 (Monotone crossing)**.** *In the transformation regime ($u > \Lambda_c$, $\sigma \geq 0$),*

$$\boxed{\dot{\xi} = \alpha\,\dot{\lambda}\,\Omega_P + (\alpha\lambda)^2, \qquad \dot{\lambda} = (\delta u - \gamma)\tanh(\kappa\sigma) + I_{\text{gate}} \geq 0,}$$

so $\dot{\xi} > 0$. Therefore $\xi = 1$ is crossed exactly once, upward, and is never an attractor.

Three regimes along the trajectory

$\xi < 1$	sub-Ricci	structure-limited; effort/perseverance,
$\xi = 1$	resonance	gift-rate = structure-rate; perfect synergy,
$\xi > 1$	super-Ricci	grace exceeds intrinsic momentum; mature Epektasis.

7 Expansion Budget and the Necessity of the Open Gate

The asymptotic Brim — hence the asymptotic curvature — is fixed by the total Sophia budget $\int_0^\infty \lambda\,dt$ (*Pleroma Christi condition*, core), via $\Omega_P(t) = \Omega_P(0) + \alpha\int_0^t \lambda\,ds$.

Proposition 4 (Expansion budget)**.** *Three asymptotic regimes:*

(1) $\int_0^\infty \lambda\,dt < \infty$: $\Omega_P \to \Omega_P^\infty < \infty$, *and* $R \to -2/(\Omega_P^\infty)^2 < 0$ *— bounded Brim, curvature plateaus.*

(2) $\lambda \to \lambda_\infty > 0$: $\Omega_P \sim \alpha\lambda_\infty t$, *and* $|R| \sim \frac{2}{(\alpha\lambda_\infty)^2} t^{-2} \to 0$ *— linear Brim.*

(3) $\lambda \to \infty$, $\mathcal{G}_{\text{recepta}}(\infty) = +\infty$: Ω_P *super-linear,* $|R| \to 0$ *faster still — unbounded Brim.*

Regime (1) is stalled Epektasis: without a sufficient Sophia budget the live domain keeps a permanent negative curvature — woundedness reduced but never healed to flatness.

Super-Ricci flattens faster than Ricci

$$\text{Ricci locus: } \Omega_P \sim \sqrt{2t} \Rightarrow |R| \sim t^{-1}; \qquad \text{super-Ricci: } \Omega_P \sim \alpha\lambda_\infty t \Rightarrow |R| \sim t^{-2}.$$

The grace-borne regime relaxes the curvature faster *than a true Ricci flow would: grace-driven Epektasis outperforms geometric self-determination at the very task — flattening — that defines Theosis.*

Why the Open Gate is needed

The closed limit $\zeta_0 = 0$ freezes the cleansed Sophia at the finite stock $\lambda_\infty = \sigma_0 + \lambda_0$, so it attains *at most* regime (2) on non-replenishable capital, and regime (3) is impossible. Sustained — and a fortiori unbounded — Brim expansion requires the bounded Open-Gate source $I_{\text{gate}} \geq 0$, the only mechanism giving $\mathcal{G}_{\text{recepta}}(\infty) = +\infty$.

$$\text{Unbounded Brim expansion} \iff \mathcal{G}_{\text{recepta}}(\infty) = +\infty \implies \text{Open Gate } (\zeta_0 > 0) \text{ required.}$$

8 Relation to the Cosmological Layer

The spatial geometry above and the Lorentzian lift share the same hyperbolic surface h_{2D} but are distinct objects. As a slice of fixed Brim, Ω_P is a scale parameter; as the cosmology, $\Omega_P(t)$ is the timelike scale factor:

$$ds^2_\Sigma = \Omega_P^2\, h_{2D} \quad \text{(spatial)}, \qquad ds^2_{\text{Vessel}} = -dt^2 + \Omega_P(t)^2\, h_{2D} \quad \text{(Lorentzian 2+1)}.$$

Their curvatures differ accordingly and do **not** contradict:

$$R_\Sigma = -\frac{2}{\Omega_P^2} \quad \text{(slice)}, \qquad R_{\text{Vessel}} = \frac{2\big(2\Omega_P\ddot{\Omega}_P + \dot{\Omega}_P{}^2 - 1\big)}{\Omega_P^2} \quad \text{(spacetime)}.$$

The slice curvature depends on Ω_P alone; the spacetime curvature also carries $\ddot{\Omega}_P$ (equivalently $\dot{\lambda}$), since the Lorentzian object records the *dynamics* of expansion, not only its instantaneous scale.

Remark. The static Riemannian cone $d\Omega_P^2 + \Omega_P^2 h_{2D}$ (with scalar $-4/\Omega_P^2$) is yet a *third* object and is **not** the Vessel spacetime: promoting Ω_P to a spacelike coordinate misassigns its dynamical role. The Brim is the flow-time here and the timelike scale factor there; any three-dimensional rendering is a visualization, not a formal model. The cosmological layer — Friedmann content, equation of state, energy conditions, eras — is developed in `vfs_cosmology.tex`.

9 Brim-Flow Geometry: Theological Reading

Interpretive readings in the framework's symbolic register; the Propositions above remain the load-bearing part. Brim-Flow is not a Ricci flow in the strict sense but a V.F.S. curvature-relaxation in which Sophia expands the Brim and the expanding Brim relaxes the curvature of the live domain. The governing chain is

$$\boxed{\sigma \longrightarrow \lambda \longrightarrow \Omega_P \longrightarrow R \longrightarrow 0^- \longrightarrow P,}$$

Sophia expands the Brim; the Brim relaxes curvature.

I. Epektasis as curvature relaxation

Since $R = -2/\Omega_P^2 < 0$ in the live domain and $\Omega_P \to \infty \Rightarrow R \to 0^-$, Theosis is not the violent abolition of curvature but its asymptotic healing. The Brim does not erase finitude; it makes finitude more spacious.

Epektasis is the asymptotic healing of wounded form through expanding receptivity.

II. Sophia as the geometry-changing fruit of transmuted resistance

Through $\dot{\Omega_P} = \alpha\lambda$, wisdom is not a static possession but an expansive principle, enlarging the vessel's capacity to receive more being, grace, and participation.

The healed wound becomes Brim: transmuted resistance becomes spatial capacity.

III. Brim as receptive space

Ω_P is not a numerical bound but the measure of receptive spaciousness. As the Brim expands the same synergic intensity occupies a smaller share of the available space, $\Omega_P \uparrow \Rightarrow r = u/\Omega_P \downarrow$, retreating from the death boundary $r \to 1^-$.

Grace does not only calm the motion; it enlarges the vessel that receives the motion.

IV. Ricci-like but not Ricci flow

Ricci flow is curvature-driven, $\partial_t g_{ij} = -2R_{ij}$; Brim-Flow is Sophia-driven, $\dot{\Omega_P} = \alpha\lambda$ with $R = -2/\Omega_P^2$. The analogy is formal and theological, not identical.

Ricci flow is curvature-driven smoothing; Brim-Flow is Sophia-driven curvature relaxation.

V. Pneumatic meaning

The Ricci-like character is a differential icon of pneumatic healing: the Spirit is not the flow, but the living source whose action — healing wounded form from within by transmutation $\sigma \to \lambda \to \Omega_P$ — the flow formally resembles.

> The Spirit heals curvature by transforming resistance into receptive spaciousness.

VI. Ricci resonance and the surpassing of nature

The resonance quantity $\xi = \alpha\lambda\Omega_P$ gives three regimes: $\xi < 1$ sub-Ricci perseverance; $\xi = 1$ Ricci resonance (grace-bearing expansion at the intrinsic rate of curvature healing); $\xi > 1$ super-Ricci Epektasis (grace outruns nature).

> Ricci flow: nature healed according to its own curvature.

> Open-Gate Brim-Flow: nature carried beyond its own curvature by grace.

Remark. By Proposition 3 the resonance $\xi = 1$ is not a resting state but a threshold crossed once, upward. The living trajectory therefore passes necessarily from self-carried ($\xi \leq 1$) to grace-carried ($\xi > 1$); keeping the gate open is the refusal to let transfiguration settle into acquired self-sufficiency (cf. Prop. 4). Maturity is not synergistic equilibrium but grace steadily outrunning nature.

VII. Resurrection as curvature jump

The reset enlarges the Brim, $\Omega_P^+ = \Omega_P^- + \kappa_R \mathcal{G}^-_{\text{recepta}}$, so $R^+ > R^-$ (both negative; $\Delta R > 0$, §5). Resurrection preserves the hyperbolic type while increasing the curvature radius: it is not a return to the previous state but a continuation of being under an expanded geometry.

> Resurrection does not erase the form; it opens the form with a greater radius.

VIII. No absolute collapse

Whenever Sophia is active, $\dot{\Omega}_P = \alpha\lambda > 0$ moves $r = u/\Omega_P$ away from the death boundary: salvation enlarges the very domain in which the trajectory can continue.

> Grace does not merely redirect the path; it enlarges the world in which the path remains possible.

IX. Final theological formulation

Being is healed neither by the elimination of motion nor by the suppression of finitude, but by the expansion of receptive capacity. Resistance becomes Sophia; Sophia expands the Brim; the Brim relaxes curvature; curvature relaxation opens the live domain to continuing participation toward Pleroma without finite completion.

Theosis is the endless expansion of receptive geometry toward Pleroma.

Brim-Flow is the Ricci-like geometry of Epektasis.

Compact Geometric Summary

barrier $B(r) \equiv$ geodesic distance to the death wall.

live domain $\cong \mathbb{H}^2, \quad R = -\dfrac{2}{\Omega_P^2}, \quad$ radius $=$ Brim.

$\dot{\Omega}_P = \alpha\lambda \implies \partial_t g = 2\frac{\alpha\lambda}{\Omega_P} g$ (forced; $=$ Ricci iff $\alpha\lambda\Omega_P = 1$).

Epektasis: $|R| \to 0$ (continuous); Resurrectio: $\Delta R > 0$ (discrete);
$\xi = \alpha\lambda\Omega_P$ crosses 1 once, upward.

V.F.S. v2.0 (Open-Gate) · Geometric Layer. Derived from the Brim law, radial barrier, and imbalance dynamics of `vfs_opengate_core.tex` and the reset / active-domain data of `vfs_opengate_lyapunov.tex`. Propositions are closed-form; the Brim-Flow theological reading is interpretive and anchored to them by explicit cross-reference.