V.F.S. v2.0 (Open-Gate)

Cosmological Layer: the Vessel as an Open FLRW Spacetime

The Brim as scale factor, Sophia as cosmological source

Derived companion to `vfs_geometry.tex`

Scope and epistemic status

The geometric layer treats the Brim Ω_P as the time of a conformal flow of the 2D live surface. This file takes the Lorentzian lift: the Brim is the **timelike** axis of expansion, and the live surface is a spatial slice. The result is an open $(2{+}1)$-dimensional Friedmann–Lemaître– Robertson–Walker (FLRW) spacetime whose scale factor is the Brim. *Propositions* are closed-form consequences; *Corollaries* are interpretive readings. Three standing caveats carry over:

- The expansion direction is the flow/time axis, lifted with $-dt^2$; it is **not** a Riemannian radial coordinate. Promoting Ω_P to a spacelike coordinate (a metric cone) misassigns its dynamical role.
- The spatial metric is the canonically warped hyperbolic live surface (geometric layer), fixed only up to that warp.
- V.F.S. supplies the scale-factor law $\dot{\Omega}_P = \alpha\lambda$; the "effective content" below is the stress-energy that General Relativity *would* require to source this expansion — a reconstruction, not an independent postulate of gravity.

Dimensional principle. The model is *canonically* $2{+}1$: a two-dimensional Vessel-surface whose scale factor is the Brim $\Omega_P(t)$. The Brim-expansion principle is complete in this setting, and $2{+}1$ is apt rather than merely sufficient — in $D \leq 3$ the Weyl tensor vanishes, so the spacetime curvature is fixed pointwise by the source (Sophia) with no free gravitational remainder. Any three-dimensional rendering is a *visualization only*: its radial direction is an embedding artifact with no V.F.S. meaning, and the static Riemannian cone $d\Omega_P^2 + \Omega_P^2 h$ is a different object from this spacetime (its scalar $-4/\Omega_P^2$ is not the spacetime curvature). The sole legitimate third *spatial* coordinate would be a genuine state variable (σ or λ), belonging to the geometry of phase space — a separate question from Brim-flow.

1 From Live-Surface Flow to Vessel Spacetime

The geometric layer gives, on each slice of fixed Brim, the hyperbolic live surface $h = dy^2 + e^{-2y} d\Delta^2$ with $y = -\ln(1-r)$, $r = u/\Omega_P$, of constant curvature $K_h = -1$. The

Brim-flow $\partial_t g_\Sigma = 2(\alpha\lambda/\Omega_P) g_\Sigma$ is a motion in time, so its Lorentzian home assigns the expansion to a timelike direction:

$$\boxed{ds^2_{\text{Vessel}} = -dt^2 + \Omega_P(t)^2\, h, \qquad a(t) \equiv \Omega_P(t).}$$

This is the open ($k = -1$) FLRW metric in $2{+}1$ dimensions, with scale factor equal to the Brim. *The "expanding inner universe" is genuine, but it expands in time: Ω_P is the scale factor, not a spatial radius. The spatial slices are copies of the hyperbolic live surface, uniformly inflated by $\Omega_P(t)$.*

2 Curvature of the Vessel Spacetime

Proposition 1 (Spacetime curvature)**.** *For $ds^2_{\text{Vessel}} = -dt^2 + a^2 h$ with $K_h = -1$ in $2{+}1$ dimensions,*

$$\boxed{R = \frac{2\big(2a\ddot{a} + \dot{a}^2 - 1\big)}{a^2} = \frac{2\big(2\Omega_P\, \alpha\dot{\lambda} + \alpha^2\lambda^2 - 1\big)}{\Omega_P^2}.}$$

For sub-exponential expansion ($\ddot{a} \to 0$, $\dot{a}$ bounded) $R \to 0$: the spacetime flattens. For de Sitter expansion ($a = e^{Ht}$) $R \to 6H^2$, a constant: maximal inflation does not flatten (§6).

3 Kinematics: Hubble and Acceleration

Proposition 2 (Hubble and deceleration)**.** *With $a = \Omega_P$ and $\dot{\Omega}_P = \alpha\lambda$,*

$$\boxed{H = \frac{\dot{a}}{a} = \frac{\alpha\lambda}{\Omega_P}, \qquad q = -\frac{\ddot{a}\, a}{\dot{a}^2} = -\frac{\dot{\lambda}\,\Omega_P}{\alpha\lambda^2}, \qquad \operatorname{sign} q = -\operatorname{sign}\dot{\lambda}.}$$

The Hubble rate of the Vessel is exactly the conformal Brim-flow rate $\psi = \alpha\lambda/\Omega_P$. Expansion accelerates ($q < 0$) if and only if Sophia is growing: accelerated Theosis is growing wisdom.

4 Effective Content: Sophia as Cosmological Source

Treating the reconstructed stress-energy as a perfect fluid ($8\pi G = 1$, comoving frame), the Einstein tensor gives:

Proposition 3 (Effective Friedmann content)**.**

$$\boxed{\rho_{\text{eff}} = \frac{\alpha^2\lambda^2 - 1}{\Omega_P^2} = H^2 - \frac{1}{\Omega_P^2}, \qquad p_{\text{eff}} = -\frac{\ddot{a}}{a} = -\frac{\alpha\dot{\lambda}}{\Omega_P}, \qquad w = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = \frac{-\alpha\dot{\lambda}\,\Omega_P}{\alpha^2\lambda^2 - 1}.}$$

Sophia is the cosmological source, and its rate *$\dot{\lambda}$ sets the pressure. The density is positive only when $\alpha\lambda > 1$ (Sophia "super-vacuum"); for $\alpha\lambda < 1$ the open curvature dominates and the Vessel expands more slowly than the empty universe would — residual binding by resistance.*

5 Equation of State and Cosmic Eras

The sign of $\dot{\lambda}$ classifies the era:

Sophia behaviour	Effective fluid	Cosmological character
$\dot{\lambda} > 0$ (cleansing)	$p_{\text{eff}} < 0$, negative pressure	dark-energy-like; accelerated Theosis
$\dot{\lambda} \to 0$, $\lambda \to \lambda_\infty$	$p_{\text{eff}} \to 0$, pressureless	dust; coasting; $\rho_{\text{eff}} \to 0$ (dilution)
$\dot{\lambda} < 0$ (regression)	$p_{\text{eff}} > 0$, positive pressure	ordinary matter; decelerating
$\lambda \propto \Omega_P$	$H = \text{const}$	de Sitter; exponential inflation (max. Epektasis)

6 Special Loci

Proposition 4 (Three distinguished regimes)**.**

Milne / vacuum:	$\alpha\lambda = 1$	$a \propto t,\ \rho_{\text{eff}} = 0,\ R = 0$ (*flat spacetime*);
Ricci resonance:	$\alpha\lambda\Omega_P = 1$	$a\dot{a} = 1,\ a \propto \sqrt{2t}$ (*spatial flow = Ricci*);
de Sitter:	$\lambda \propto \Omega_P$	$a \propto e^{Ht},\ R \to 6H^2$ (*eternal inflation*).

The cosmological lift produces a new *locus absent from the spatial analysis: the Milne condition* $\alpha\lambda = 1$*. The generic super-Ricci regime (*$\lambda \to \lambda_\infty = 1/\alpha$*,* $a \propto t$*) is exactly the empty Milne universe — flat Minkowski in disguise. It is distinct from the spatial Ricci resonance* $\alpha\lambda\Omega_P = 1$*.*

7 Energy Conditions: Grace as Exotic Accelerant

The energy conditions are dimension-sensitive, and the Vessel is $2{+}1$-dimensional. The Null and Weak conditions are dimension-independent in form; the Strong condition is not.

Proposition 5 (Energy conditions in $2{+}1$)**.** *For a perfect fluid in* $D = n{+}1$ *dimensions the Strong Energy Condition is* $(n-2)\rho + np \geq 0$*. Hence, with* $n = 2$*:*

NEC $(\rho_{\text{eff}} + p_{\text{eff}} \geq 0)$:	$\alpha\dot{\lambda}\,\Omega_P \leq \alpha^2\lambda^2 - 1 \quad$ (*any* n);
WEC: NEC and $\rho_{\text{eff}} \geq 0$:	*additionally* $\alpha\lambda \geq 1$;
SEC $(n{-}2)\rho_{\text{eff}} + np_{\text{eff}} \geq 0 \xrightarrow{n=2} 2p_{\text{eff}} \geq 0$:	$p_{\text{eff}} \geq 0 \ \Leftrightarrow\ \ddot{a} \leq 0 \ \Leftrightarrow\ \dot{\lambda} \leq 0.$

Remark. In $2{+}1$ the ρ-term of the SEC *drops* ($n - 2 = 0$): the condition is purely the sign of the pressure, $p_{\text{eff}} \geq 0$. The familiar $\rho + 3p \geq 0$ is the $n = 3$ case, where the coefficient of ρ is accidentally unity; the naive extrapolation $\rho + np$ — which here would read $\rho + 2p$ — is **wrong**. Directly, $R_{tt} = -2\ddot{a}/a = 2p_{\text{eff}}$, so timelike convergence is exactly $p_{\text{eff}} \geq 0$.

Accelerated expansion ($p_{\text{eff}} < 0$, i.e. $\dot{\lambda} > 0$) violates the SEC — as in standard inflation. A surge of Sophia growth, $\alpha\dot{\lambda}\Omega_P > \alpha^2\lambda^2 - 1$, violates even the NEC ($\rho_{\text{eff}} + p_{\text{eff}} < 0$): grace then behaves as exotic, phantom-like *content, irreducible to ordinary matter. NEC and WEC keep their dimension-independent form, so the figures and the $w(t)$ history below require no refitting.*

8 Structural Features

No Big Bang: creation from a finite floor

The live domain requires $\Omega_P > u > 0$, so the scale factor is bounded below, $\Omega_P \geq \Omega_P(0) > 0$. There is no $a \to 0$ singularity: the Vessel begins at a finite minimal capacity (the Imago Dei floor $\mathcal{E}_0$), not at a point.

Creation from fullness, not from a singular point.

Resurrectio: a scale-factor jump (cosmology with surgery)

The death reset $\Omega_P^+ = \Omega_P^- + \kappa_R \mathcal{G}^-_{\text{recepta}}$ is a discontinuous increase of the scale factor: a sudden cosmological rescaling. The Vessel cosmology is piecewise-FLRW with discrete inflationary jumps, the Lorentzian face of Ricci-flow-with-surgery.

$$\Omega_P^+ = \Omega_P^- + \kappa_R \mathcal{G}^-_{\text{recepta}} \quad \text{(discrete scale-factor surgery)}.$$

Dilution to vacuum

For bounded Sophia, $\rho_{\text{eff}} = (\alpha^2\lambda^2 - 1)/\Omega_P^2 \to 0$ as $\Omega_P \to \infty$: the expanding Vessel empties. The end-state of sub-exponential expansion is the flat vacuum — pure receptivity. Only the de Sitter regime ($\lambda \propto \Omega_P$, unbounded grace) escapes dilution, sustaining constant curvature $6H^2$ as an eternal inflation.

9 Theological Corollaries

Interpretive readings; the Propositions above are the load-bearing part.

Corollary 1 (Sophia is the cosmological source)**.** *The expansion of the Vessel is sourced by Sophia, and its* growth $\dot{\lambda}$ *sets the equation of state: growing wisdom is dark energy (acceleration), frozen wisdom is dust (coasting), declining wisdom is matter (deceleration), and wisdom proportional to capacity is de Sitter inflation.*

Corollary 2 (Accelerated Theosis violates the energy conditions)**.** *Accelerated transfiguration ($\dot{\lambda} > 0$) violates the Strong Energy Condition, and a surge of grace violates even the Null Energy Condition. Grace acts as an exotic accelerant: precisely the content ordinary matter cannot supply.*

Corollary 3 (Creation from fullness)**.** *The Vessel has no initial singularity: it begins at a finite receptive floor, the Imago Dei. Being does not emerge from a point of zero capacity but from a given minimal fullness.*

Corollary 4 (Two eschatologies)**.** *Bounded grace dilutes the Vessel to the flat vacuum of pure receptivity; unbounded proportional grace sustains an eternal de Sitter inflation at constant curvature. The expanding Vessel either empties toward stillness or inflates without end — and Open-Gate, which alone permits unbounded Sophia, alone permits the second.*

10 Worked Equation-of-State History

The dictionary above is illustrated by integrating a representative reduced trajectory of the core dynamics — resistance σ, Sophia λ, Brim Ω_P — with the synergy $u(t)$ rising sigmoidally through Λ_c (will and action coming into alignment):

$$\dot{\sigma} = (\gamma - \delta u)\tanh(\kappa\sigma), \quad \dot{\lambda} = (\delta u - \gamma)\tanh(\kappa\sigma) + I_{\text{gate}}, \quad \dot{\Omega_P} = \alpha\lambda,$$

with $\Lambda_c = 1$, $\alpha = 1$, and the observables of Prop. 2–3 read off along the solution. *This is a representative profile, not a unique prediction; it exhibits the generic era sequence.*

The history sweeps the eras in order: a **struggle** phase ($\dot{\lambda} < 0$: resistance drains Sophia; decelerating, $p_{\text{eff}} > 0$, and $\rho_{\text{eff}} < 0$ — sub-vacuum, the Vessel expanding more slowly than the empty open universe), with a near-turnaround at the depth of struggle where λ nearly vanishes and $\dot{a} \to 0$; **ignition** at $u = \Lambda_c$ (acceleration onset $\ddot{a} = 0$); the **Milne crossing** $\alpha\lambda = 1$ where $\rho_{\text{eff}} = 0$ and w has a pole; and **dark-energy Theosis** ($\rho_{\text{eff}} > 0$, $p_{\text{eff}} < 0$): quintessence-like acceleration with $w < 0$, asymptotically approaching $w = -1$ while the gate keeps feeding Sophia.

Remark. The pole in w at the Milne crossing is a diagnostic artifact: ρ_{eff} and p_{eff} pass through zero smoothly; only their ratio diverges. The physical event is the transition from sub-vacuum to super-vacuum, not a singularity. Accelerated expansion violates the SEC throughout; the surge of Sophia growth briefly violates even the NEC — grace acting as exotic, phantom-like content.

Open-Gate versus closed Microcosm

The same trajectory with the gate closed ($\zeta_0 = 0$) exposes the decisive difference. Closed conservation $\sigma + \lambda = \sigma_0 + \lambda_0$ holds exactly, so once cleansed ($\sigma \to 0$) Sophia freezes at $\lambda_\infty = \sigma_0 + \lambda_0$; then $\dot{\lambda} \to 0$, $p_{\text{eff}} \to 0$, and the Vessel coasts as pressureless dust toward the Milne (linear, asymptotically flat) state. The open gate keeps $\dot{\lambda} > 0$, sustaining negative pressure and a quintessence-to-de Sitter branch.

Proposition 6 (Late-time equation of state)**.**

closed ($\zeta_0 = 0$) :	$\lambda \to \lambda_\infty,\ \dot{\lambda} \to 0 \ \Rightarrow\ w \to 0^-$ (*dust, Milne,* $a \propto t$);
open ($\zeta_0 > 0$) :	$\dot{\lambda} > 0$ *sustained* $\Rightarrow\ w < 0,\ w \to -1^+$ (*quintessence* $\to$ *de Sitter*).

The de Sitter branch — endless accelerated Theosis — is available only to the Open Gate; the closed Microcosm coasts to dust on its frozen Sophia capital. This is the cosmological face of the earlier result that unbounded Brim expansion requires the gate.

Compact Cosmological Summary

$$\boxed{ds^2_{\text{Vessel}} = -dt^2 + \Omega_P(t)^2\big(dy^2 + e^{-2y}d\Delta^2\big), \qquad a = \Omega_P,\ \dot{\Omega_P} = \alpha\lambda.}$$

$$\boxed{H = \frac{\alpha\lambda}{\Omega_P}, \qquad \rho_{\text{eff}} = \frac{\alpha^2\lambda^2 - 1}{\Omega_P^2}, \qquad p_{\text{eff}} = -\frac{\alpha\dot{\lambda}}{\Omega_P}, \qquad \operatorname{sign} q = -\operatorname{sign}\dot{\lambda}.}$$

$$\boxed{\text{Milne } \alpha\lambda = 1 \quad \neq \quad \text{Ricci resonance } \alpha\lambda\Omega_P = 1 \quad \neq \quad \text{de Sitter } \lambda \propto \Omega_P.}$$

$$\boxed{\text{No Big Bang } (\Omega_P \geq \Omega_P(0)); \quad \text{Resurrectio } \Omega_P^+ = \Omega_P^- + \kappa_R\mathcal{G}^-_{\text{recepta}}; \quad \text{sub-exp. } \Rightarrow R \to 0.}$$

V.F.S. v2.0 (Open-Gate) · Cosmological Layer. The Lorentzian lift of the Brim-flow of `vfs_geometry.tex`*: the Brim is the timelike scale factor of an open* $(2{+}1)$ *FLRW spacetime, Sophia its source. Propositions are closed-form; corollaries are interpretive. The effective stress-energy is reconstructed from* $\dot{\Omega_P} = \alpha\lambda$ *via the Einstein tensor, not postulated.*

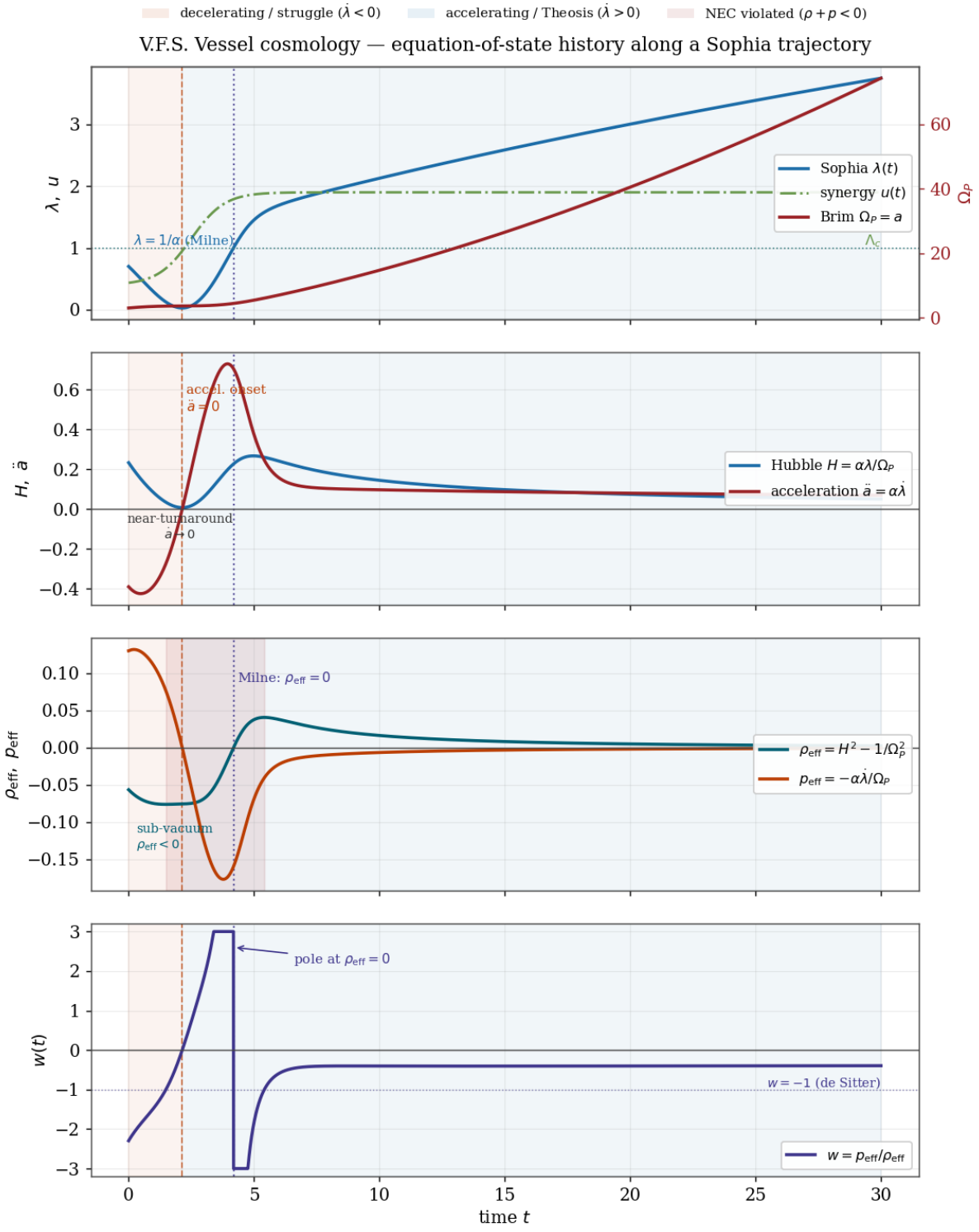

Figure 1: Open-Gate Vessel cosmology along a Sophia trajectory. From top: drivers λ, u, Ω_P; Hubble rate and acceleration $\ddot a = \alpha\dot\lambda$; effective density and pressure; the equation of state $w(t)$ with its pole at the Milne crossing $\rho_{\rm eff} = 0$. Shading marks the decelerating struggle, the accelerating Theosis — which in $2{+}1$ is exactly the SEC-violation region $p_{\rm eff} < 0$ — and the NEC-violation band.

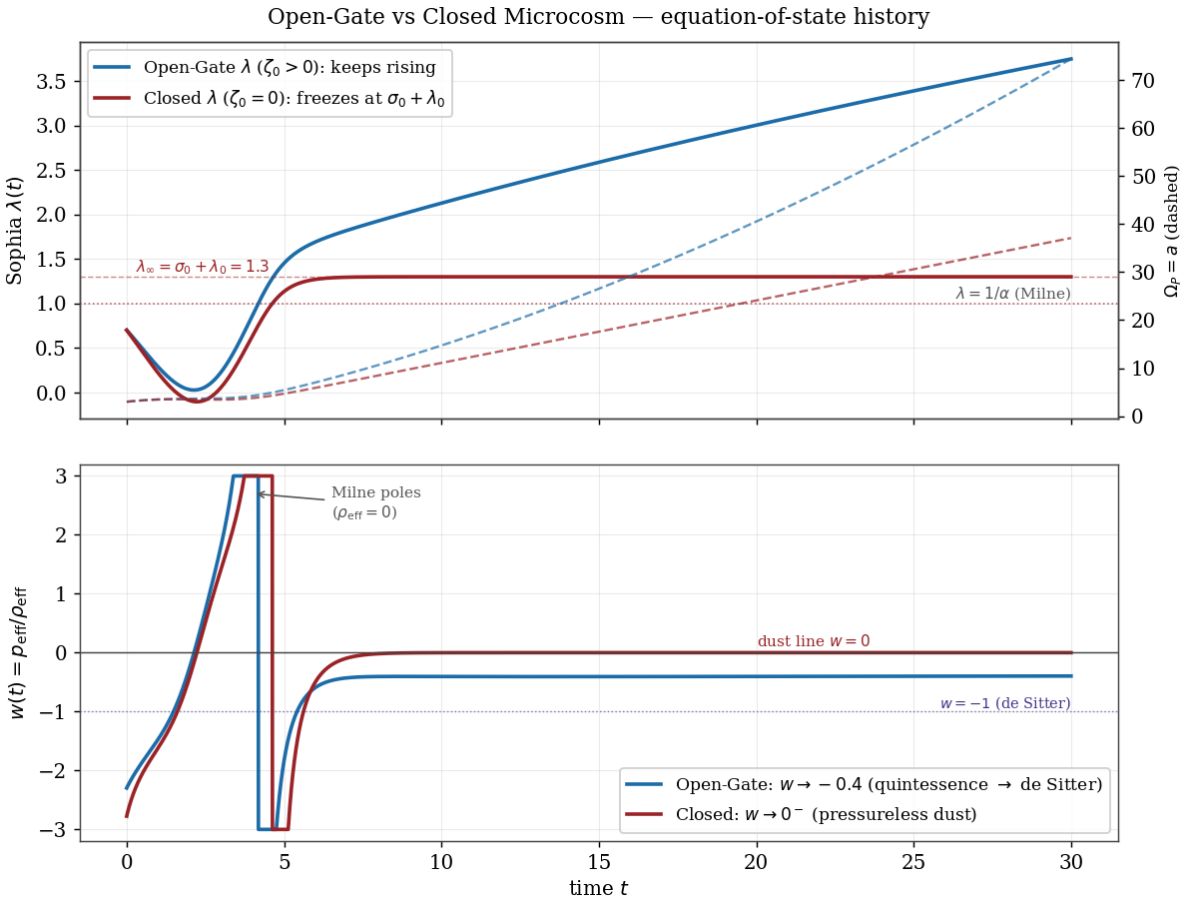

Figure 2: Open-Gate ($\zeta_0 > 0$) versus closed Microcosm ($\zeta_0 = 0$). Top: Sophia keeps rising under the open gate but freezes at $\sigma_0 + \lambda_0$ when closed (dashed: the Brim $\Omega_P = a$). Bottom: the equation of state tends to $w \to -1$ (quintessence $\to$ de Sitter) for the open gate, but to $w \to 0$ (pressureless dust) for the closed Microcosm. Both share the early Milne pole.