V.F.S. 3.6

One Vessel, One Lorentzian Interior,
Two $\mathbb{Z}_2$ Folding Phases

v15 — softplus-consistent closing audit of V.F.S. 3.6

8 August 2026 (v15)

Status. Closing edition v15 of 3.6. Does not modify frozen 2.0, 3.0, 3.5, 4.0, 4.5, 4.6.

Every statement carries an explicit status: [imported], [derived], [proposed], [conditional], [numerical], [legacy numerical], [open/exported].

3.6 is closed as an *interface theory*. The exact typing $G_{\text{reset}} \neq G_{\text{recepta}}$, the junction invariant I_R, the hybrid coordinate Θ_R, the ledger identities, the reduced canonical fibres, the reset-cost bounds, the projection typing, the gauge results and the spatial interface results do **not** depend on the numerical implementation of the Sophia positive-part convention.

The preserved computational checkpoint v12 used the archived module `vessel2.py`, which implements $\lambda_+^{\text{code}} = \max(\lambda, 0)$, whereas frozen Core 2.0 uses the smooth softplus $\lambda_+^{\text{fr}} = \frac{1}{m}\ln(1+e^{m\lambda})$. Hence the v12 merger/KKT/multistart/continuation numbers carry the status [legacy numerical / hard-part engine] in closing v15; their softplus revalidation is an explicit regression obligation, not a premise of the exact results of 3.6.

Explicit $\mathcal{J}^{\theta}_{KW}$, the lift $X^{\text{junction}}_{3.0} \to X^{\text{union}}_{\text{core}}$ and rigorous coupled backward uniqueness remain [open/exported].

Abstract

3.6 introduces no new base geometry, no new folding dynamics and no new entities. It is an *interface theory*: it reconciles, within one Vessel, the Lorentzian interior of 3.0, the forced $\mathbb{Z}_2$ folding of 2.0, and the Riemannian state space of 3.5. Literal signature change is not part of 3.6; the absolute-sign sector is closed as a global $\mathbb{Z}_2$ gauge, and spatial wall morphology is not identified with the cathartic neck. Active 3.6 retains the constitutional strict interior reserve

$$\sigma_0 + \lambda_0 < C_\sigma, \qquad R_c := C_\sigma - \sigma_0 - \lambda_0 > 0,$$

which together with the frozen grace window yields a uniform positive reset cost and one-sided reachable-set exclusion.

The closing edition v15 preserves the canonical event datum $G_{\text{reset}} \neq G_{\text{recepta}}$, where G_{reset} is the local datum of a particular junction and G_{recepta} is the continuous cumulative ledger. Canonical Resurrectio preserves

$$I_R := \lambda + \lambda_{\text{form}} - \Omega_P, \qquad I_R^+ = I_R^-,$$

but I_R is not a dwell first integral. Instead the hybrid coordinate $\Theta_R := \sigma_L + I_R - G_{\mathrm{recepta}}$ is continuous through $\mathfrak{R}_{3.0}$ and satisfies $\dot{\Theta}_R = -\alpha\lambda$ on smooth Core 2.0 dwell arcs. The Reduced Canonical Fibre Theorem reduces scalar merger to equality of $(\sigma_L, G_{\mathrm{recepta}}, I_R)$; for $\kappa_R > 0$ and no additional upper bound on G_{reset}, event-feasibility is always constructively solvable.

The preserved computational checkpoint v12 was obtained from the *archived* smooth numerical module `vessel2.py` in the current cumulative-ledger convention. A later source audit established that this implementation uses $\lambda_+^{\mathrm{code}} = \max(\lambda, 0)$ in the gate saturation, whereas the frozen Core 2.0 convention is $\lambda_+^{\mathrm{fr}} = \frac{1}{m}\ln(1 + e^{m\lambda})$.

The reported parameter-locked values

$$a_0^* \approx 1.3991246, \quad q_A^* = 0, \quad q_B^* \approx 0.1109261, \qquad \|\Delta H_K\|_2 \approx 0.0381403693,$$

together with the KKT residual of order 10^{-9}, the positive definite projected Hessian, the active-bound release, the wide multistart, the history/event continuation and the tolerance certification (tight residual $1.61 \cdot 10^{-15}$, condition number $9.16 \cdot 10^4$), are retained as a *legacy hard-part-engine computational checkpoint*. They demonstrate the numerical stability of the archived implementation, but they are not a numerical verification of the frozen softplus dynamics.

Closing v15 therefore separates two levels: **the exact interface algebra is closed**, whereas **the frozen-softplus merger numerics remain pending regression**. This does not alter the exact canonical junction algebra, is not a frozen reset law, is not an analytic global-minimum theorem, and does not create a full-$X_{\mathrm{core}}^{\mathrm{union}}$ temporal non-injectivity theorem.

Contents

1 Ontology

Axiom 3.6 1 (One Vessel, one interior)**.** The Vessel is one Lorentzian interior $(\mathcal{M}_L, g_L)$ of the frozen volume 3.0. Folding does not create a second manifold, a second Vessel, and does not generate a plurality of "rooms". [proposed]

Folding endows one Vessel with two morphological phases related by the symmetry $q_{\text{fold}} \mapsto -q_{\text{fold}}$. However, the frozen folding field does not live as a scalar on the whole $3{+}1$ manifold $\mathcal{M}_L$, but on a family of living spatial surfaces. Introduce their worldvolume

$$\mathcal{S}_{\text{live}} := \bigcup_{\chi} \{\chi\} \times \Sigma_\Omega(\chi), \qquad q_{\text{fold}} : \mathcal{S}_{\text{live}} \longrightarrow \mathbb{R}.$$

Then

$$\mathcal{P}_+ = \{(\chi, x) \in \mathcal{S}_{\text{live}} : q_{\text{fold}}(\chi, x) > 0\}, \qquad \mathcal{P}_- = \{(\chi, x) \in \mathcal{S}_{\text{live}} : q_{\text{fold}}(\chi, x) < 0\},$$

$$\Sigma_{\text{fold}} = \{(\chi, x) \in \mathcal{S}_{\text{live}} : q_{\text{fold}}(\chi, x) = 0\}, \qquad \mathcal{S}_{\text{live}} = \mathcal{P}_+ \cup \Sigma_{\text{fold}} \cup \mathcal{P}_-.$$

The Lorentzian Vessel remains one $(\mathcal{M}_L, g_L)$; two-phaseness is a morphology of its living spatial slices, not a decomposition of $\mathcal{M}_L$ into two Lorentzian manifolds.

Remark 1.1 (terminology)**.** We distinguish strictly:

$$\text{domain} \neq \text{phase worldvolume} \neq \text{room} \neq \text{Vessel}.$$

For fixed χ introduce the slice phases

$$\mathcal{P}_+(\chi) = \{x \in \Sigma_\Omega(\chi) : q_{\text{fold}}(\chi, x) > 0\}, \qquad \mathcal{P}_-(\chi) = \{x \in \Sigma_\Omega(\chi) : q_{\text{fold}}(\chi, x) < 0\}.$$

A *domain* is a connected component of $\mathcal{P}_+(\chi)$ or $\mathcal{P}_-(\chi)$ on a particular living surface. By contrast $\mathcal{P}_+, \mathcal{P}_- \subset \mathcal{S}_{\text{live}}$ are their phase worldvolumes in time. A *room* is the whole Lorentzian room, i.e. one Vessel. The "two rooms" language of earlier editions is discarded.

2 Import from the frozen folding layer

3.6 does not re-prove anything below.

2.1 Forced activation

[imported] For the form potential $\mathcal{E}(q) = -\frac{1}{2}Aq^2 + \frac{1}{4}Bq^4$, $B > 0$, the gradient flow $\dot{q} = Aq - Bq^3$ gives: $q = 0$ is stable $\iff A < 0$; $q_\pm = \pm\sqrt{A/B}$ are stable $\iff A > 0$. After the transition $A > 0$ the old form $q = 0$ becomes a repeller.

Remark 2.1. Hence folding is neither a choice nor an event that "ought to have extinguished". It is the forced loss of stability of the old form. The claim of earlier working packages about "non-self-extinguishing folding" is withdrawn as a description of the normal operation of the pitchfork.

2.2 The pitchfork is structurally forced

[imported] Under $\mathbb{Z}_2$ symmetry, a one-dimensional centre manifold and transversality, the pitchfork normal form is a consequence of the reduction, not an assumption.

2.3 Field and domain walls

[imported]

$$\partial_t q_{\text{fold}} = A_{\text{fold}} q_{\text{fold}} - B_{\text{fold}} q_{\text{fold}}^3 + \frac{D_q}{\Omega_P^2} \Delta_h q_{\text{fold}}$$

— the Allen–Cahn equation on an expanding hyperbolic surface. A static wall has profile $q_* \tanh(\xi/(\sqrt{2}\,\ell))$ with

$$\ell_{\text{phys}} = \sqrt{D_q/A_{\text{fold}}} \quad (\text{independent of } \Omega_P), \qquad \tau_{\text{wall}} \sim \sqrt{D_q}\, A_{\text{fold}}^{3/2}/B_{\text{fold}}.$$

The network moves by mean curvature, $v_n = -M\kappa$, $M \sim D_q/\Omega_P^2$, and coarsens; the hyperbolic geometry ($K_h = -1$) accelerates coarsening.

Proposition 2.2 (Finite coarsening budget)**.** *[imported] The frozen folding addendum already gives a finite "activity budget"*

$$I_{\text{bud},\infty} = \int_0^\infty \frac{dt}{\Omega_P(t)^2} < \infty,$$

which simultaneously bounds diffusive smoothing and domain-wall motion. For the characteristic comoving scale one has a finite limiting change:

$$\ell_{\text{com}}^2(\infty) = \ell_{\text{com}}^2(0) + 2cD_q I_{\text{bud},\infty} < \infty.$$

Hence the comoving network really does "freeze" after the finite coarsening budget is spent; late growth of the physical scale is predominantly expansion, not unbounded re-coarsening.

Remark 2.3. Wall thickness and tension are sign-independent and enter the scalar energetics. This does not make every network detail a global invariant, but the finite coarsening budget is already a frozen theorem-level constraint and needs no new proof in 3.6.

2.4 Typing of the living surface: genus as a sector label

Frozen volume 3 takes the living surface to be a compact hyperbolic quotient Σ_γ, $\gamma \geq 2$, with comoving area $A_h = 4\pi(\gamma - 1)$ by Gauss–Bonnet, under the heading "Setting and honesty flags" — that is, as a finiteness condition for the Perelman functionals.

Proposition 2.4 (Genus does not determine the surface)**.** *A hyperbolic structure on Σ_γ is a point of Teichmüller space $\mathcal{T}_\gamma$ of dimension $6\gamma - 6$, which is strictly positive for $\gamma \geq 2$. Hence the genus alone does not fix the living surface:*

γ	χ	A_h	$\mathcal{W}^{\max}$	$\dim H^1(\Sigma;\mathbb{Z}/2)$	$\dim \mathcal{T}_\gamma$
2	-2	12.566	$+0.307$	4	6
3	-4	25.133	-0.386	6	12
4	-6	37.699	-0.792	8	18
5	-8	50.265	-1.079	10	24
10	-18	113.097	-1.890	20	54

[derived, standard]

Proposition 2.5 (Audit of the dependence of the 3.6 results on Σ)**.** *Every theorem-level result of 3.6 depends on the living surface* exclusively *through* $\chi(\Sigma_\gamma) = 2 - 2\gamma$*:*

- *Theorem 7.4: only compactness without boundary is needed (integration by parts);*
- *Proposition 6.3 and* $[\Sigma_{\text{fold}}] = 0$*: these depend on* $H^1(\Sigma; \mathbb{Z}/2) = (\mathbb{Z}/2)^{2\gamma}$*, i.e. on the genus;*
- $A_h = 4\pi(\gamma - 1)$*: a consequence of Gauss–Bonnet at curvature* -1*;*
- *the normalization of* $\mathcal{Q}_\Sigma$*: division by* $\text{Area}_h(\Sigma_\gamma)$*.*

By contrast, the following depend on the full *moduli point: geodesic lengths, the systole, stationarity of noncontractible walls, the coarsening rate, and — through the wall fraction — the value of* $\mathcal{Q}_\Sigma$ *for two-phase states. All of these already belong to the open Question 2.* *[derived/audit]*

Remark 2.6 (dependence of $\mathcal{Q}_\Sigma$ on the surface)**.** [numerical] For a homogeneous field $\mathcal{Q}_\Sigma$ is a mean and does not depend on the area. For a two-phase configuration $\mathcal{Q}_\Sigma \approx q_*^2(1 - f_{\text{wall}})$ with $f_{\text{wall}} = L_{\text{wall}}\ell/A_h$, so it depends on the network length relative to A_h — that is, on the actual surface, not on the genus alone.

Axiom 3.6 2 (Genus as a fixed topological sector label)**.** γ is typed as a *fixed topological sector label* of the Vessel: a constant for a given Vessel, but **not** a constitutional datum alongside E_0. Grounds:

(i) the frozen corpus introduces γ as a finiteness condition, not as a constitutional property; 3.6 has no licence to promote an honesty flag to constitutional status;

(ii) by Proposition 2.4 the genus underdetermines the surface: a constitutional datum would have to be a point of $\mathcal{T}_\gamma$, not a number;

(iii) by Proposition 2.5 the weaker typing suffices for all proved results of 3.6.

[proposed/typing]

Remark 2.7 (measurability is not constitutionality)**.** A_h enters the entropy maximum $\mathcal{W}^{\max} = 1 - \ln(A_h/2\pi)$, so the genus is *observable* from the Vessel's own entropy. But observability does not establish constitutional status: E_0 is constitutional because frozen 2.0 calls it "the Imago-Dei reserve", i.e. through a textual ground, not through measurability. No such ground exists for γ. The earlier argument "measurable $\Rightarrow$ constitutional" is withdrawn.

Remark 2.8 (indicator for the next stage)**.** If a future edition requires constitutional status for the living surface, it will need a full moduli point, not a genus. The indicator that this will happen is concrete: the first statement depending on geodesic lengths — i.e. closing Question 2 — will require more than χ.

3 Two geometric registers

Proposition 3.1 (Type separation of 3.0 and 3.5)**.** *3.0 and 3.5 describe different geometric objects:* $(\mathcal{M}_L, g_L)$ *is a Lorentzian interior evolution, whereas*

$$(X_R, g_R), \qquad g_R = \frac{d\sigma_R^2 + du^2 + d\lambda^2}{\sigma_R^2},$$

is a Riemannian state-space representation. Therefore the scheme $L \to R \to L$ in 3.6 means a change of the geometric description *of the state, not a proven change of signature of one metric tensor.* *[derived/imported]*

Remark 3.2. 3.6 neither needs nor asserts a theorem on the impossibility of signature-changing geometry. Such a question lies outside its scope: it suffices that frozen 3.0 and 3.5 already have different domains and different geometric roles for the coordinates.

4 The interface π_R

4.1 State vectors: union state and canonical junction state

Definition 4.1 (Cross-layer union state)**.** The wide state which 3.6 uses for the interface between frozen layers is denoted

$$X_{\text{core}}^{\text{union}} := \big(V, F, \sigma_L, \lambda, \lambda_{\text{form}}, \Omega_P, H_K, W, G_{\text{recepta}}, H_R;\ q_{\text{fold}}(\cdot)\big).$$

For compatibility with earlier sections, $X_{\text{core}} \equiv X_{\text{core}}^{\text{union}}$. These are union coordinates of different frozen namespaces, not a claim that every frozen reset map acts on all of them. [typed]

Definition 4.2 (Canonical 3.0 junction state)**.** For the physical interior junction of 3.0 we single out only that active block of 3.6 for which frozen 3.0/4.0 provides canonical junction typing:

$$X_{3.0}^{\text{junction}} := \big(\sigma_L, \lambda, \lambda_{\text{form}}, \Omega_P, G_{\text{recepta}}, H_K, W;\ q_{\text{fold}}(\cdot)\big).$$

The coordinates V, F, H_R remain in $X_{\text{core}}^{\text{union}}$ but are not automatically included in the domain/codomain of $\mathfrak{R}_{3.0}$: H_R belongs to the raw $\mathfrak{R}_{\text{core2}}$ namespace, and explicit canonical 3.0 jump laws for V, F are not established in the present frozen audit. [imported/typed]

Remark 4.3 (Auxiliary 3.0 junction variables)**.** Frozen 3.0 also has geometric/ascent quantities such as the ascent scale A and a scalar/slaved folding amplitude. In the current 3.6 they are not added as new independent coordinates of $X_{\text{core}}^{\text{union}}$: A is a geometric junction output/slaved quantity, and the scalar folding amplitude is lifted below to a spatial-field relation.

$$X_R = (\sigma_R, u, \lambda), \qquad g_R = \frac{d\sigma_R^2 + du^2 + d\lambda^2}{\sigma_R^2} \ \cong\ \mathbb{H}^3.$$

3.0 geometrizes the core dynamics but does not make all geometric quantities algebraic functions of the current (σ_L, λ). In particular

$$\frac{d\Omega_P}{d\chi} = \alpha\lambda$$

makes Ω_P a genuine state/history variable; derived geometric quantities such as $A(\Omega_P)$ may be slaved to it. This distinction must be preserved in the interface.

Definition 4.4 (Projection)**.** For $X \in X_{\text{core}}^{\text{union}}$:

$$\pi_R : X_{\text{core}}^{\text{union}} \to X_R, \qquad \pi_R(X) = \big(\sigma_R,\ \sqrt{VF + \varepsilon},\ \lambda\big).$$

At the level of interface identification, $\sigma_R := \sigma_L$. [derived]

Remark 4.5. In this definition the domain of π_R is precisely $X_{\text{core}}^{\text{union}}$. This does not mean that canonical $\mathfrak{R}_{3.0}$ must be extended to all coordinates of $X_{\text{core}}^{\text{union}}$ before its own reset fibres on $X_{3.0}^{\text{junction}}$ can be investigated.

4.2 Classification of information

Variable	Status	Ground
σ_L	preserved	$\sigma_R = \sigma_L$; only the geometric type changes
λ	preserved	a coordinate of X_R without transformation
ε	preserved	constant; floor $u \geq \sqrt{\varepsilon}$
V, F	projected	only through u; individually invisible
u	derived/independent	derived in core, independent coordinate in X_R
G_{recepta}	forgotten/conditional	not in X_R; recoverable only given the arc constant and extra data
$\lambda_{\text{form}}, \Omega_P, H_K, W, H_R$	forgotten	not in X_R
$q_{\text{fold}}(\cdot), Q_{\text{fold}}(\cdot)$	forgotten	a field on the living surface, not a point of X_R

Proposition 4.6 (The (V, F) subfibre is one-dimensional and non-compact)**.** *For fixed (σ_R, u_0, λ) with $c := u_0^2 - \varepsilon > 0$ one has*

$$\pi_R^{-1}(\sigma_R, u_0, \lambda)\big|_{(V,F)} = \{(V, F) \in \mathbb{R}^2_{>0} :\ VF = c\}.$$

This is a one-dimensional non-compact hyperbola. The condition $u_0 < \Omega_P$ constrains the value of u_0 relative to Ω_P, but for fixed u_0 it gives no separate upper bounds on V and F: the branch $V \to \infty$, $F = c/V \to 0^+$ remains on the same subfibre. *[derived]*

Corollary 4.7 (One parameter for the (V, F) subfibre only)**.** *The (V, F) part of the hidden information can be parametrized by $\delta = \ln(V/F) \in \mathbb{R}$. This does not mean that the full fibre of π_R is one-dimensional: it additionally contains all core variables and field data which the projection does not carry to X_R.*

Proposition 4.8 (The fibre contracts)**.** *For $\Delta = V - F$ frozen 2.0 gives $\dot{\Delta} = -(\mu a + \rho)\Delta$. Hence $\Delta \to 0$ exponentially: the fibre information is not merely hidden, it is rapidly destroyed. Numerically, alignment completes within $\sim 0.1\%$ of the arc duration.* *[derived]*

Remark 4.9 (consequence for 4.7)**.** A reader of the fibre must read *early* in the arc, or read the fact of alignment itself rather than the current value.

4.3 The inverse map

Proposition 4.10 ($\iota_{R\to L}$ does not exist as a map)**.** *π_R forgets $\Omega_P, \lambda_{\text{form}}, H_K, W, H_R$ and the whole field q_{fold}. None of these is recoverable from (σ_R, u, λ). Hence no single-valued inverse map $X_R \to X_{\text{core}}$ exists.* *[derived]*

Definition 4.11 (Admissible continuation)**.** Instead of $\iota_{R\to L}$ we introduce a continuation with additional data:

$$\iota : X_R \times \mathcal{S} \longrightarrow X_{\text{core}}, \qquad \mathcal{S} = \big(\Omega_P, \lambda_{\text{form}}, H_K, W, H_R, G_{\text{recepta}}, q_{\text{fold}}(\cdot), \delta\big),$$

where δ parametrizes the fibre (Proposition 4.6). The consistency condition is $\pi_R \circ \iota = \mathrm{id}_{X_R}$. [proposed]

Remark 4.12. $\mathcal{S}$ is a working list of the data invisible to 3.5. The mere fact of their invisibility to X_R does not yet require a nonlocal reader: a full local core-reader may see these quantities directly. For 4.7 only information which is not recoverable even from the full current X_{core} will be relevant.

5 Ledger and local kairos arcs

Frozen 2.0 gives, on each continuous life arc, the balance

$$\frac{d}{dt}(\sigma_L + \lambda + \lambda_{\text{form}}) = I_{\text{gate}}, \qquad \dot{G}_{\text{recepta}} = I_{\text{gate}},$$

and it explicitly warns that through Resurrectio "books are re-opened" and each arc has its own integration constant. Therefore 3.6 does not introduce a physical reset law for G_{recepta} which is absent from the chosen frozen reset map, but distinguishes physical bookkeeping from the local chart coordinate.

Definition 5.1 (Local increment of received grace)**.** Let t_n^+ be the entry time of the n-th life arc. Define

$$\widehat{G}_n(t) := \int_{t_n^+}^{t} I_{\text{gate}}(s)\, ds.$$

If a particular implementation uses a global cumulative variable $G_{\text{recepta}}(t)$, then $\widehat{G}_n(t) = G_{\text{recepta}}(t) - G_{\text{recepta}}(t_n^+)$. By definition $\widehat{G}_n(t_n^+) = 0$. This is a coordinate rebase, not a new physical reset law. [derived/interface]

Theorem 5.2 (First integral of a local arc)**.** *On the n-th continuous arc the quantity*

$$C_n = \sigma_L + \lambda + \lambda_{\text{form}} - \widehat{G}_n$$

is constant. Its value equals the entry budget $C_n = \sigma_{L,n}^+ + \lambda_n^+ + \lambda_{\text{form},n}^+$. This is precisely the local-arc integration constant; frozen 2.0 permits $C_{n+1} \neq C_n$ through Resurrectio. *[derived from frozen balance]*

Remark 5.3 (global and local bookkeeping)**.** If one implementation keeps a global cumulative G_{recepta}, then the global identity on a given arc can be rewritten through $\widehat{G}_n$. If another frozen convention opens a new arc-book with its own zero of reception, then $\widehat{G}_n$ simply coincides with its local $G_{\text{recepta},n}$. 3.6 must not mix these conventions.

Definition 5.4 (Local kairos arc)**.** The n-th arc carries the chart $(\sigma_{L,n}, \chi_n, C_n, \widehat{G}_n)$. At the end of the arc the chart closes; the next one may be rebased with $\chi_{n+1} = 0$, $\widehat{G}_{n+1} = 0$, but the physical state passes through the frozen hybrid map, not through an artificial zeroing of core variables.

Proposition 5.5 (Linear-metabolism Resurrectio specialization)**.** *In the folding/Lyapunov layer used here for the interface test, the frozen map is*

$$\sigma^+ = q_R \sigma^-, \qquad \lambda^+ = \lambda^- + (1 - q_R)\sigma^-,$$

$$\Omega_P^+ = \Omega_P^- + \kappa_R G_{\text{recepta}}^-, \qquad (V,F)^+ = (V,F)^-, \qquad u^+ = u^-,$$

with $0 \le q_R < 1$, $\kappa_R > 0$. The frozen folding extension leaves q_{fold} continuous/inert across this reset: $q_{\text{fold}}^+ = q_{\text{fold}}^-$. We denote precisely this specialized map by $\mathfrak{R}_{\text{LM}}$. This map contains no rule $H_K^+ = 0$ and no licence to replace it by an artificial reset $V = F = 0$, $\lambda = 0$, $H_K = 0$, and so on. *[imported]*

5.1 Reset atlas: three frozen levels

Definition 5.6 (Three distinct reset/junction maps)**.** 3.6 does not use the word "Resurrectio" as the name of one universal algebraic map. The frozen corpus contains at least three distinct levels:

$$\mathfrak{R}_{\mathrm{core2}} \neq \mathfrak{R}_{\mathrm{LM}} \neq \mathfrak{R}_{3.0}.$$

Their status in 3.6 is:

(i) $\mathfrak{R}_{\mathrm{core2}}$ — the raw/general Core 2.0 re-entry with re-seeding of V, F, Ω_P and an event-level datum Γ_R;

(ii) $\mathfrak{R}_{\mathrm{LM}}$ — the Lyapunov/linear-metabolism specialization, which 3.6 uses for the grace window and the reachable-set theorem;

(iii) $\mathfrak{R}_{3.0}$ — the canonical 3.0 interior junction, which frozen 4.0 directly imports as the physical junction for the Vessel's history.

No algebraic identity from one map transfers to another without a separate interface theorem. [imported/typed]

Proposition 5.7 (Raw Core 2.0 is an event-conditioned map)**.** *In raw Core 2.0 the resurrection grace Γ_R enters as a datum/mode of the event itself; the frozen corpus does not specify a constitutive law $\Gamma_R = \Gamma_R(X^-)$. Hence the correct type of the map is $\mathfrak{R}_{\mathrm{core2}}(\,\cdot\,;\Gamma_R)$, or $\widehat{\mathfrak{R}}_{\mathrm{core2}} : (X^-, \Gamma_R) \mapsto X^+$. In particular, the question "on which state coordinates does Γ_R depend?" is not a frozen functional-dependence problem unless a new constitutive closure is separately added.* *[imported/typed]*

Proposition 5.8 (Canonical 3.0 interior junction)**.** *In the current 3.6 notation, the frozen 3.0/4.0 junction fixes a local reset reception datum G_{reset} and the partition*

$$\sigma_L^+ = \sigma_L^-, \qquad \Omega_P^+ = \Omega_P^- + \kappa_R G_{\mathrm{reset}},$$

$$\lambda^+ = \lambda^- + m_R, \qquad \lambda^+_{\mathrm{form}} = \lambda^-_{\mathrm{form}} + \Delta_{\mathrm{emb}},$$

where

$$\kappa_R G_{\mathrm{reset}} = m_R + \Delta_{\mathrm{emb}}, \qquad 0 \le \Delta_{\mathrm{emb}} \le \kappa_R G_{\mathrm{reset}}.$$

For the frozen ascent scale, $\dfrac{A^+}{A^-} = \exp\!\left(\dfrac{\kappa_R G_{\mathrm{reset}}}{E_0}\right)$*.*

Here $G_{\mathrm{reset}} \neq G_{\mathrm{recepta}}$ by type: G_{reset} is the local event/reset datum of a particular junction, not a transferable stock and not a cumulative state coordinate; G_{recepta} is the global cumulative ledger. The frozen corpus does not specify a constitutive closure $G_{\mathrm{reset}} = F(X^-)$. Frozen 4.0 separately fixes $G^+_{\mathrm{recepta}} = G^-_{\mathrm{recepta}}$.

This canonical junction is denoted $\mathfrak{R}_{3.0}$. Its natural current-3.6 state space is $X^{\mathrm{junction}}_{3.0}$, not the whole cross-layer $X^{\mathrm{union}}_{\mathrm{core}}$. *[imported/typed]*

Remark 5.9 (Form memory and the scalar folding chart)**.** Canonical $\mathfrak{R}_{3.0}$ already closes the jump law for form memory: $\lambda^+_{\mathrm{form}} - \lambda^-_{\mathrm{form}} = \Delta_{\mathrm{emb}}$. In the frozen slaved folding chart one also meets the scalar relation $q^+_{\mathrm{fold}} = q(\lambda^+)$. It must not be read pointwise as $q^+_{\mathrm{fold}}(x) = q(\lambda^+)$ on the spatial 3.6 field: that would erase the domain/wall morphology which the scalar chart does not type.

Definition 5.10 (Gauge-invariant spatial amplitude functional)**.** On the compact living surface Σ_γ define

$$\boxed{\mathcal{Q}_\Sigma[q] := \frac{1}{\text{Area}_h(\Sigma_\gamma)} \int_{\Sigma_\gamma} q(x)^2 \, dA_h.}$$

The functional is invariant under the global gauge $q \mapsto -q$ and measures the sign-blind mean-square folding amplitude. [derived/typed]

5.2 ODE↔PDE coupling: the zero-order additive class

Item (B1) of the backward-uniqueness programme requires fixing how the spatial folding field enters the scalar Sophia budget. Since λ and λ_{form} are Vessel-level scalars while q_{fold} is a field on Σ_γ, the coupling must return a scalar functional of the field.

Frozen homogeneous Core 2.0 determines only the *value* of such a functional on spatially constant fields. By itself this does **not** exclude gradient-dependent or nonlocal terms. Therefore 3.6 explicitly considers the minimal *zero-order additive class*

$$\mathcal{F}[q] = \frac{1}{\text{Area}_h(\Sigma_\gamma)} \int_{\Sigma_\gamma} f(q(x)) \, dA_h,$$

and it is within this class that exact homogeneous reduction determines the coupling uniquely.

Theorem 5.11 (Uniqueness within the zero-order additive class)**.** *Let*

$$\mathcal{F}[q] = \frac{1}{\text{Area}_h(\Sigma_\gamma)} \int_{\Sigma_\gamma} f(q(x)) \, dA_h$$

for some measurable $f : \mathbb{R} \to \mathbb{R}$, and suppose that for every spatially constant field $q \equiv q_0$ the exact reduction

$$\mathcal{F}[q] = q_0^2$$

holds. Then $f(s) = s^2$ for all $s \in \mathbb{R}$, and hence

$$\boxed{\mathcal{F}[q] = \mathcal{Q}_\Sigma[q] = \frac{1}{\text{Area}_h(\Sigma_\gamma)} \int_{\Sigma_\gamma} q(x)^2 \, dA_h}$$

Thus $\mathcal{Q}_\Sigma$ is the unique coupling functional within the zero-order additive class*.* *[derived/conditional on coupling class]*

Proof. For a constant field $q \equiv q_0$ we have $\mathcal{F}[q] = \frac{1}{\text{Area}_h} \int_{\Sigma_\gamma} f(q_0) \, dA_h = f(q_0)$. Exact homogeneous reduction gives $f(q_0) = q_0^2$ for every $q_0 \in \mathbb{R}$. □

Remark 5.12 (limit of the theorem)**.** Theorem 5.11 does **not** assert uniqueness among all possible local or nonlocal spatial functionals. For instance, the formally admissible functional

$$\mathcal{Q}_\Sigma[q] + c \cdot \frac{1}{\text{Area}_h} \int_{\Sigma_\gamma} |\nabla_h q|^2 \, dA_h$$

also has exact homogeneous reduction, because the gradient term vanishes on constant fields. Excluding such higher-order couplings requires a separate interface axiom or a physical derivation.

Corollary 5.13 (Gauge invariance of the zero-order coupling)**.** *Within the zero-order additive class Theorem 5.11 gives $f(-s) = f(s)$, hence $\mathcal{F}[-q] = \mathcal{F}[q]$. So the adopted scalar Sophia coupling $\mathcal{Q}_\Sigma$ is invariant under the global transformation $q \mapsto -q$. [derived within adopted coupling class]*

Definition 5.14 (Adopted coupled dwell system of 3.6)**.** Within the minimal zero-order additive coupling class, 3.6 uses

$$\dot{\lambda} = \lambda^{(0)} - \eta_{\text{fold}} \mathcal{Q}_\Sigma[q_{\text{fold}}]\lambda, \qquad \dot{\lambda_{\text{form}}} = \eta_{\text{fold}} \mathcal{Q}_\Sigma[q_{\text{fold}}]\lambda,$$

$$\partial_t q_{\text{fold}} = A_{\text{fold}}(\lambda, \dot{\lambda}^{(0)})\, q_{\text{fold}} - B_{\text{fold}} q_{\text{fold}}^3 + \frac{D_q}{\Omega_P^2} \Delta_h q_{\text{fold}},$$

while the remaining scalar equations stay frozen Core 2.0 unchanged. [proposed/typed; exact homogeneous extension]

Proposition 5.15 (Exact homogeneous reduction of the coupled system)**.** *If $q_{\text{fold}}(\chi, \cdot)$ is spatially constant, then $\Delta_h q_{\text{fold}} = 0$ and $\mathcal{Q}_\Sigma[q_{\text{fold}}] = q_{\text{fold}}^2$, so Definition 5.14 coincides with the frozen scalar system of 2.0 identically, not approximately. [derived]*

Remark 5.16 (periodic discretization regression witness)**.** [numerical] To check the exact homogeneous reduction, a periodic 48×48 finite-difference discretization was used with a homogeneous initial field $q_{\text{fold}} \equiv 0.10$ and `rtol`$= 10^{-11}$ on $T = 8$. Since a homogeneous field has $\Delta_h q_{\text{fold}} = 0$ regardless of the spatial geometry, this regression test checks precisely the ODE↔PDE homogeneous coupling and *not* the geometric properties of a particular hyperbolic surface.

component	coupled discretization	frozen ODE	|difference|
λ	1.246131954476	1.246131954475	$1.4 \cdot 10^{-12}$
λ_{form}	2.249348955429	2.249348955424	$4.9 \cdot 10^{-12}$
Ω_P	4.016358163528	4.016358163527	$6.6 \cdot 10^{-13}$
σ_L	0.000000025654	0.000000025654	$1.1 \cdot 10^{-15}$
q_{fold}	0.528302556274	0.528302556273	$7.5 \cdot 10^{-13}$

The field remained homogeneous to machine precision. Witness: `coupling.py`.

Proposition 5.17 (B5: Lipschitz estimate for the spatial coupling)**.** *Let $|q_{\text{fold}}^A|, |q_{\text{fold}}^B| \le M$ on a fixed finite interval, where M may be taken from Corollary 5.28, and $w := q_{\text{fold}}^A - q_{\text{fold}}^B$. Then*

$$\mathcal{Q}_\Sigma[q_{\text{fold}}^A] - \mathcal{Q}_\Sigma[q_{\text{fold}}^B] = \langle (q_{\text{fold}}^A + q_{\text{fold}}^B)\, w \rangle,$$

and by Cauchy–Schwarz

$$\boxed{|\mathcal{Q}_\Sigma[q_{\text{fold}}^A] - \mathcal{Q}_\Sigma[q_{\text{fold}}^B]| \le 2M \, \|w\|_{L^2(h)}}$$

*So the algebraic item **(B5)** of Question 3 is closed with the explicit finite-arc constant $2M$. [derived]*

Remark 5.18 (numerical check)**.** [numerical] For six configurations with $M = 1.2$ the ratio $|\Delta \mathcal{Q}_\Sigma| / (2M\|w\|)$ takes the values 0.3234, 0.3347, 0.3532, 0.3788, 0.3084, 0.3428. The bound is respected in every tested configuration and is not excessively coarse.

5.3 Sophia sign, B_{min} and the lower Brim bound

The estimates of Propositions 12.9 and 5.17 require a uniform lower bound $B_{\text{fold}} \geq B_{\text{min}} > 0$. Separately, item (B2) requires $\Omega_P \geq \Omega_{\text{min}} > 0$. These two tasks **must be distinguished**: the lower bound on Ω_P belongs to the frozen admissible state domain, whereas the lower bound on B_{fold} depends on control of the sign of Sophia.

Remark 5.19 (frozen softplus and the absence of automatic $\lambda \geq 0$)**.** Frozen Core 2.0 contains no general theorem $\lambda \geq 0$. The gate saturation uses the *smooth* positive part

$$\lambda_+ = \frac{1}{m}\ln\bigl(1 + e^{m\lambda}\bigr), \qquad m > 0,$$

precisely because λ as a dynamical coordinate is not typed a priori as non-negative. In particular

$$\boxed{\lambda_+(0) = \frac{\ln 2}{m} > 0}$$

so at the boundary $\lambda = 0$ the Sophia saturation of the gate does **not** vanish.

The ledger identity by itself does not determine the sign either: $\lambda = C_n + \widehat{G}_n - \sigma_L - \lambda_{\text{form}}$. [imported/typing]

Proposition 5.20 (Softplus-correct sufficient condition for invariance of $\{\lambda \geq 0\}$)**.** *At the boundary $\lambda = 0$ the folding drain $-\eta_{\text{fold}}\mathcal{Q}_\Sigma[q_{\text{fold}}]\lambda$ vanishes, but the gate saturation remains finite because $\lambda_+(0) = \ln 2/m$. Hence*

$$\boxed{\dot{\lambda}\big|_{\lambda=0} = (\delta u - \gamma)\tanh(\kappa\sigma_L) + \frac{\zeta_0\, g_{\text{eff}}\, e^{-\phi\sigma_L}}{1 + \chi_{\text{sat}}\dfrac{\ln 2}{m}}}$$

Manere gives $u \geq \sqrt{\varepsilon}$, and the term $(\delta u - \gamma)\tanh(\kappa\sigma_L)$ is nondecreasing in u for $\sigma_L \geq 0$; so the worst admissible case is $u = \sqrt{\varepsilon}$. A sufficient condition for positive invariance of $\{\lambda \geq 0\}$ is

$$\boxed{\frac{\zeta_0\, g_{\text{eff}}\, e^{-\phi\sigma_L}}{1 + \chi_{\text{sat}}\dfrac{\ln 2}{m}} \;\geq\; (\gamma - \delta\sqrt{\varepsilon})\tanh(\kappa\sigma_L)}$$

for all admissible σ_L on the arc. *[derived/conditional]*

Remark 5.21 (softplus-correct critical resistance)**.** Write

$$\boxed{s_m := \left(1 + \chi_{\text{sat}}\tfrac{\ln 2}{m}\right)^{-1}, \qquad 0 < s_m < 1.}$$

The critical resistance for fixed g_{eff} is given implicitly by

$$\zeta_0\, g_{\text{eff}}\, s_m\, e^{-\phi\sigma_{\text{crit}}} = (\gamma - \delta\sqrt{\varepsilon})\tanh(\kappa\sigma_{\text{crit}}).$$

The earlier numerical table 1.46921, 1.11427, 0.76005, 0.40366 was obtained for the hard-positive-part convention $\lambda_+ = \max(\lambda, 0)$, i.e. formally for $s_m = 1$. It is **not a canonical numerical table** for the frozen softplus system and is removed from the theorem-level text in closing v15.

Since $s_m < 1$, softplus reduces the effective gate input at the boundary $\lambda = 0$, so the softplus-correct σ_{crit} is strictly smaller than the hard-part value, other things equal. The numerical value is not fixed in 3.6 until the smoothing parameter m is supplied from the frozen parameter set. [derived; numerical value deferred]

Remark 5.22 (relation to the earlier manageability boundary)**.** Proposition 5.20 has the same structural form: the gate support must compensate the maximal adverse work term at $\lambda = 0$. However, an exact identification with the earlier "room-manageability boundary" is permitted only if that boundary was derived from the *same* softplus-correct inequality, i.e. with the same factor s_m.

The numerical agreement of hard-part values is not by itself a proof of an algebraic identity. Hence 3.6 does not assert a new identity without a separate source-level check of the frozen manageability derivation. [audit/conditional]

Corollary 5.23 (B_{min} under non-negative Sophia)**.** *Suppose the condition of Proposition 5.20 holds on the arc and the entry datum has $\lambda(t_0) \geq 0$. Then positive invariance gives $\lambda(t) \geq 0$ on the whole arc, and since $B_{\text{fold}} = b + 2c_1\eta_{\text{fold}}\lambda$,*

$$\boxed{B_{\text{fold}} \geq b =: B_{\text{min}} > 0.}$$

[derived/conditional]

Proposition 5.24 (B2: constitutional Brim floor)**.** *The frozen admissible state domain already imposes $\Omega_P \geq E_0 > 0$. Hence for any two admissible trajectories in the coupled backward-uniqueness problem one may take $\Omega_{\text{min}} := E_0$, and the coefficient $a(y) = D_q/\Omega_P^2$ satisfies the uniform estimate*

$$0 < a(y) \leq \frac{D_q}{E_0^2}.$$

*Thus item **(B2)** of Question 3 is closed* without *assuming $\lambda \geq 0$.*

The condition $\lambda \geq 0$ plays a different role: it gives $\dot{\Omega}_P = \alpha\lambda \geq 0$, i.e. forward nondecrease of Brim along the arc — but this is not the source of the constitutional lower bound. [imported/derived]

Remark 5.25 (state-domain floor versus dynamical monotonicity)**.** $\Omega_P \geq E_0$ is a restriction on the admissible state domain of the frozen theory. By contrast $\dot{\Omega}_P = \alpha\lambda$ is a dynamical statement: for $\lambda < 0$ the smooth vector field may have $\dot{\Omega}_P < 0$; that is a question of preservation of the admissible domain, not of the existence of a lower bound *on the class of admissible solutions* on which the backward-uniqueness problem is posed. So B2 uses the imported domain floor, while the Sophia sign controls forward Brim monotonicity; the two statements are not to be conflated.

5.4 Finiteness of $A_{\max}$ on an arc

Proposition 5.26 (Explicit finite-arc chain for $A_{\max}$)**.** *Let an admissible arc have duration T, entry budget C_n, initial Brim $\Omega_P(t_0)$, and suppose $\lambda \geq 0$ on it. Then*

$$\begin{aligned}
&\textit{(L1)} \quad \lambda \leq C_n + \widehat{G}_n \leq C_n + \zeta_0 T =: \lambda_{\max}, \\
&\textit{(L2)} \quad \Omega_P \leq \Omega_P(t_0) + \alpha\lambda_{\max} T =: \Omega_P^{\max}, \\
&\textit{(L3)} \quad u < \Omega_P \leq \Omega_P^{\max}, \\
&\textit{(L4)} \quad |\dot{\sigma}_L| = |\gamma - \delta u|\,|\tanh(\kappa\sigma_L)| \leq \gamma + \delta\Omega_P^{\max}, \\
&\textit{(L5)} \quad \dot{\lambda}^{(0)} = -\dot{\sigma}_L + I_{\text{gate}} \leq \gamma + \delta\Omega_P^{\max} + \zeta_0, \\
&\textit{(L6)} \quad A_{\text{fold}} \leq c_0\lambda_{\max} + c_1\big(\gamma + \delta\Omega_P^{\max} + \zeta_0\big) - a_0 =: A_{\max}.
\end{aligned}$$

(L1) uses $\lambda = C_n + \widehat{G}_n - \sigma_L - \lambda_{\text{form}}$ together with $\sigma_L \geq 0$, $\lambda_{\text{form}} \geq 0$, $\dot{\widehat{G}}_n = I_{\text{gate}} \leq \zeta_0$. Hence on every fixed finite arc an explicit finite $A_{\max}$ exists; it depends on T and is not a uniform asymptotic bound. *[derived/conditional]*

Remark 5.27 (the estimate is crude and non-uniform in T)**.** [numerical] For the tested parameter set the finite-arc chain substantially overestimates the actual maximum of A_{fold} and grows roughly quadratically in T, whereas the tested trajectory settles on a bounded plateau:

T	bound $\lambda_{\max}$	actual λ	bound $A_{\max}$	actual max A_{fold}
4	6.450	1.720	9.69	1.0815
8	12.450	1.720	19.38	1.0815
20	30.450	1.720	62.85	1.0815
60	90.450	1.720	363.75	1.0815

This is not a defect of the bound: its job is to supply a finite constant on a fixed $[0, T]$, sufficient for local backward-uniqueness estimates. The chain does not prove $\sup_{t\geq 0} A_{\text{fold}}(t) < \infty$ and is not used as an asymptotic theorem.

Corollary 5.28 (Explicit finite-arc field bound)**.** *Let $A_{\text{fold}}(t) \leq A_{\max}$, $B_{\text{fold}}(t) \geq B_{\min} > 0$ on a fixed finite arc, and put $A_+ := \max\{A_{\max}, 0\}$. For two trajectories set*

$$\boxed{M := \max\Big\{\|q^A_{\text{fold}}(t_0)\|_{L^\infty},\ \|q^B_{\text{fold}}(t_0)\|_{L^\infty},\ \sqrt{A_+/B_{\min}}\Big\}.}$$

Then by Proposition 12.9 both fields remain in $\{\|q_{\text{fold}}\|_{L^\infty} \leq M\}$ on the whole arc, and by Corollary 5.23 one may take $B_{\min} = b$.

Moreover $\mathcal{Q}_\Sigma[q_{\text{fold}}] \leq M^2$, so the Form–Brim exchange rate $r(t) = \eta_{\text{fold}}\mathcal{Q}_\Sigma[q_{\text{fold}}]/\alpha$ satisfies

$$\boxed{0 \leq r(t) \leq r_{\max} := \frac{\eta_{\text{fold}} M^2}{\alpha} < \infty}$$

on every fixed finite arc. Hence the finite $r_{\max}$ of Theorem 13.10 is no longer a separate assumption — given finite initial L^∞ data and the hypotheses of Proposition 5.26 and Corollary 5.23. *[derived/conditional]*

Remark 5.29 (status of the backward-uniqueness programme after v15)**.** Of items (B1)–(B6):

- **(B1)** the adopted coupling is fixed within the zero-order additive class by Theorem 5.11 and Definition 5.14. This is not a claim of uniqueness among higher-gradient or nonlocal couplings.
- **(B2)** closed on the admissible state domain by Proposition 5.24: $\Omega_P \geq E_0 =: \Omega_{\text{min}} > 0$.
- **(B5)** closed by Proposition 5.17; the explicit constant M on a fixed finite arc is given by Corollary 5.28.

Still open: **(B3)** Lipschitz control $|A_{\text{fold}}(y^A) - A_{\text{fold}}(y^B)| + |a(y^A) - a(y^B)| \leq C|z|$; **(B4)** sufficient field regularity, e.g. $q^B_{\text{fold}} \in L^\infty(0, T; H^2)$; **(B6)** an appropriate log-convexity / Carleman estimate for the exact coupled difference system.

Proposition 5.26 supplies finite coefficient bounds on fixed intervals, but boundedness of the coefficients is **not** by itself (B3).

Axiom 3.6 3 (Minimal spatial lift of the canonical scalar reset)**.** 3.6 interprets the frozen scalar reset not as a pointwise field law but as a constraint on the post-junction gauge-invariant amplitude:

$$\mathcal{Q}_\Sigma[q^+_{\text{fold}}] = q(\lambda^+)^2.$$

That is, the canonical scalar of 3.0 determines the *amount* of folding amplitude but does not select the spatial morphology. For nonzero amplitude one may write

$$q^+_{\text{fold}}(x) = a^+ \varphi^+(x), \qquad a^+ := \sqrt{\mathcal{Q}_\Sigma[q^+_{\text{fold}}]} = |q(\lambda^+)|,$$

with the normalization

$$\frac{1}{\text{Area}_h(\Sigma_\gamma)} \int_{\Sigma_\gamma} (\varphi^+)^2 \, dA_h = 1, \qquad \varphi^+ \sim -\varphi^+.$$

No selection law for $[\varphi^+]$ from frozen 3.0 is added here. [proposed/typing bridge]

Definition 5.30 (Canonical spatial junction relation)**.** Because of the unresolved (H_K, W) jump law and the undetermined post-junction morphology, the canonical spatial junction in 3.6 is more correctly typed as a relation

$$X^+ \in \mathfrak{R}^{\text{spatial}}_{3.0}\big(X^-; G_{\text{reset}}, m_R\big), \qquad X^\pm \in X^{\text{junction}}_{3.0},$$

where the exact frozen constraints are given by Proposition 5.8, the spatial field satisfies Axiom 3, and (H^+_K, W^+) have frozen reset-write status without an imported algebraic formula. [typed]

5.5 Admissibility constraints on $\mathcal{J}^\theta_{KW}$

3.6 does not derive the canonical gate-memory law and has no frozen ground to do so. But it can *narrow the class* of admissible laws using only what is already proved. Below are four constraints with explicit statuses, and one falsifiability thesis that makes the conditional candidate testable.

Proposition 5.31 (J1: the absorbing floor is preserved)**.** *Frozen Core 2.0 calls $H_K = 0$ an absorbing floor (*Filtrum Tetelestai*): at $H_K = 0$ the loss term vanishes, so $\dot{H}_K \geq 0$.*

This is a structural property of the memory, not merely of the flow link. Hence any admissible $\mathcal{J}^{\theta}_{KW}$ must satisfy

$$H_K^+ \geq 0, \qquad W^+ \geq 0.$$

[imported]

Proposition 5.32 (J2: global $\mathbb{Z}_2$ gauge invariance)**.** *Since the absolute sign of the folding field is a global gauge label (Proposition 6.4), an admissible canonical gate-memory relation must not depend on the choice of representative q or $-q$. Hence*

$$\boxed{\mathcal{J}^{\theta}_{KW}(Y^-; E) = \mathcal{J}^{\theta}_{KW}(\mathsf{g}Y^-; E)}$$

where g acts as $q_{\text{fold}}(\chi, x) \mapsto -q_{\text{fold}}(\chi, x)$ for all x and leaves the remaining gauge-invariant prestate data unchanged.

This is a constraint on the global sign *only. It does **not** mean that $\mathcal{J}^{\theta}_{KW}$ may see the spatial field exclusively through $\mathcal{Q}_\Sigma$: other gauge-invariant morphology functionals are not excluded here.* *[derived/typing]*

Proposition 5.33 (J3: null-event consistency)**.** *At $G_{\text{reset}} = 0$ the canonical $\mathfrak{R}_{3.0}$ partition gives $m_R = \Delta_{\text{emb}} = 0$. Hence the exact frozen* scalar canonical block *is the identity:*

$$\sigma_L^+ = \sigma_L^-, \quad \lambda^+ = \lambda^-, \quad \lambda^+_{\text{form}} = \lambda^-_{\text{form}}, \quad \Omega_P^+ = \Omega_P^-, \quad G^+_{\text{recepta}} = G^-_{\text{recepta}}.$$

*This does **not** automatically establish identity on the unresolved gate-memory coordinates or on the spatial morphology.*

For (H_K, W), 3.6 separately adopts the natural null-event consistency:

$$\boxed{G_{\text{reset}} = 0 \implies H_K^+ = H_K^-, \quad W^+ = W^-.}$$

This is a proposed consistency requirement on a future $\mathcal{J}^{\theta}_{KW}$, not an imported frozen jump formula. *[proposed/consistency]*

Proposition 5.34 (J4: proposed reception-Lipschitz bound)**.** *For quantitative regularity of a future gate-memory relation, 3.6 proposes a local linear-growth condition: there exists $C_{KW} \geq 0$ such that*

$$\boxed{\left\|(H_K^+, W^+) - (H_K^-, W^-)\right\| \leq C_{KW}\, \kappa_R G_{\text{reset}}}$$

on the stated admissible junction domain.

This is a stronger *regularity assumption than J3 itself: J3 requires only zero write at a zero event, whereas J4 additionally imposes at-most-linear scaling near the null event. J4 does not fix the sign of the write and does not determine a constitutive law.* *[proposed/regularity]*

Remark 5.35 (what 3.6 deliberately does not add)**.** Monotone opening $H_K^+ \geq H_K^-$ is not postulated. The frozen dwell flow permits H_K to decrease in the corresponding backward-transmissive regime, and for the jump there is no frozen warrant either for monotone increase or for monotone decrease.

Nor is $H_K^+ = 0$, $W^+ = 0$ postulated, and no universal memory-only constitutive law is frozen.

Post-v12 diagnostics showed only that simple memory-only laws do not reproduce the *whole tested family* of optimized merger-required outputs. That is a discrimination result for a numerical required-write family, not a physical refutation of every possible canonical memory-only law.

5.6 Falsifiability: what the archived hard-part candidate requires

Proposition 5.36 (The archived hard-part candidate requires an opposite-sign write)**.** *For the archived v12 hard-part-engine candidate,*

$$H_K^- = (0.5589638927,\ 0.7318285776), \qquad H_K^+ = (0.5859368076,\ 0.7048628655),$$

hence $\Delta H_K = (+0.0269729149,\ -0.0269657121)$ *with* $\sum_i \Delta H_{K,i} = +7.20 \cdot 10^{-6}$.

So that archived implementation candidate *requires writes of opposite sign on the two branches, almost exactly antisymmetric. Relative magnitudes:* $+4.83\%$ *and* -3.68%*. This remains a useful falsifiability diagnostic for any future law proposed specifically to reproduce the archived candidate, but it is not yet a frozen-softplus constitutive requirement.* *[legacy numerical]*

Corollary 5.37 (Conditional falsifiability relative to the archived candidate)**.** *Any gate-memory law imposing* $H_K^+ \geq H_K^-$ *at every junction is incompatible with the* archived hard-part-engine candidate*, since its branch B requires* $\Delta H_{K,B} < 0$*.*

*This is a sharp compatibility test for reproducing that archived candidate. It is **not** a theorem that the frozen-softplus dynamics must itself realize the same required pair.* *[derived from legacy numerical candidate]*

Remark 5.38 (Two-point fit of the archived required write)**.** [legacy numerical] For the archived hard-part candidate only, the required write is exactly interpolated by $H_{K,i}^+ = H_{K,i}^- + c\,(H^* - H_{K,i}^-)$ with $H^* = 0.6454077771$, $c = 0.3120280295$ (residual $\leq 5 \cdot 10^{-17}$), and H^* lies strictly between the two pre-values.

This is merely a two-point description of the archived numerical candidate. It is neither a derivation of $\mathcal{J}_{KW}^{\theta}$ nor a prediction for the frozen softplus system.

Open Question 1 (Canonical gate-memory jump law)**.** The frozen gate interface classifies H_K and W as variables which may be written/jump at a private reset, but the current audit yielded no canonical explicit algebraic formula. The correct type of a future law must distinguish the structural parameters θ of the system from the local event datum E:

$$(H_K^+, W^+) \in \mathcal{J}_{KW}^{\theta}(Y^-; E), \qquad E = (G_{\text{reset}}, m_R).$$

Therefore 3.6 does not invent a reset $H_K \mapsto 0$, $W \mapsto 0$ or any other selection law.

After v15 the admissible design space is narrowed: J1 gives a positivity requirement, J2 global $\mathbb{Z}_2$ gauge invariance, J3 proposed null-event consistency, and J4 optional proposed reception-Lipschitz regularity. Corollary 5.37 separately gives a sharp compatibility test for the archived hard-part candidate.

Post-v12 constitutive diagnostics of the *archived hard-part family* show that the required optimized outputs of that numerical family do not support a simple universal memory-only rule. These diagnostics characterize the archived required-write family and are **not** constitutive evidence for frozen-softplus Core 2.0 prior to softplus trajectory revalidation. The correct typing of the unresolved law does not depend on which numerical engine is used.

Explicit $\mathcal{J}^{\theta}_{KW}$ therefore remains the unresolved algebraic law of the active $X^{\text{junction}}_{3.0}$ and is exported beyond 3.6. [open/exported]

Remark 5.39 (Scope of the current 3.6)**.** The reachable-set and grace-window results below belong to $\mathfrak{R}_{\text{LM}}$. The canonical temporal junction problem is first formulated on $X^{\text{junction}}_{3.0}$ through the relation $\mathfrak{R}^{\text{spatial}}_{3.0}$. Raw $\mathfrak{R}_{\text{core2}}$ is retained as a general re-entry layer and a structural comparison object, but its kernel is not the kernel of the canonical temporal junction. Any claim about the full $X^{\text{union}}_{\text{core}}$ requires a separate lift/interface result.

Axiom 3.6 4 (Strict interior grace reserve)**.** For active 3.6 admissibility the initial grace budget lies strictly inside the frozen ceiling:

$$\sigma_0 + \lambda_0 < C_\sigma, \qquad \text{hence} \qquad R_c := C_\sigma - \sigma_0 - \lambda_0 > 0.$$

This imposes no lower bound on the current σ_L and in particular does not forbid the canonical terminus limit $\sigma_L \to 0$. The condition is added only as a constitutional admissibility condition of 3.6; frozen 2.0 is not rewritten. [proposed/constitutional]

Proposition 5.40 (Uniform reception floor at Resurrectio)**.** *The frozen grace window for the linear-metabolism Resurrectio has the lower bound*

$$G^{-}_{\text{recepta}} \;\geq\; \max\{R_c,\; R^{\text{ceil}}_{\text{min}}\}, \qquad R^{\text{ceil}}_{\text{min}} = \frac{(1-q_R)\sigma^{-}}{q^{*}\kappa_R}.$$

Together with Axiom 4 this gives the history-independent bound

$$G^{-}_{\text{recepta}} \;\geq\; g_{\text{min}} := R_c > 0$$

for every admissible transition in the chosen linear-metabolism layer. Hence each reset has a minimal Brim jump $\Delta\Omega^{\text{reset}}_{P,\text{min}} = \kappa_R R_c > 0$. *[derived from frozen grace-window + Axiom 4]*

Remark 5.41 (why no σ_{min} is introduced)**.** The state-dependent term $R^{\text{ceil}}_{\text{min}}$ may tend to zero together with $\sigma^{-} \to 0$. The uniform floor comes not from forbidding this limit but from the strictly positive reserve R_c. That is exactly why the cleansed/terminus regime of 3.0 remains available.

Remark 5.42 (scope of the reset map)**.** All grace-window and reachable-set statements of this section refer precisely to $\mathfrak{R}_{\text{LM}}$. They are not formulas of raw $\mathfrak{R}_{\text{core2}}$ or canonical $\mathfrak{R}_{3.0}$; the reset atlas of Definition 5.6 is a mandatory typing rule for all subsequent references to Resurrectio.

Axiom 3.6 5 (A chart reset is not a new time)**.**

$$\text{local chart reset} \;\neq\; \text{creation of a new global time}.$$

The global hybrid history remains one; the local rebase $\chi_{n+1} = 0$ and $\widehat{G}_{n+1} = 0$ is a change of chart, whereas the physical state passes through the frozen reset map of Proposition 5.5. [proposed interface rule]

6 Status of the sign of q_{fold}

Proposition 6.1 (F2a: the scalar economy does not see the sign)**.** *The scalar folding economy enters the budgets through $Q_{\text{fold}} = q_{\text{fold}}^2$. Hence the local states $+q_*$ and $-q_*$ have the same scalar folding magnitude and are not distinguished by laws depending only on Q_{fold}. This is a statement about a sign-blind scalar economy, not about unobservability of the sign by a full core-reader.* [derived]

Proposition 6.2 (The sign is not carried through π_R)**.** *The field $q_{\text{fold}}(\cdot)$ does not enter X_R either with or without sign. Hence π_R is not the place where the sign quotient arises; the factorisation $q_{\text{fold}} \mapsto Q_{\text{fold}} = q_{\text{fold}}^2$ already happens in the scalar folding economy.* [derived]

Proposition 6.3 (F2b: no topological sign monodromy)**.** *Under the frozen reading, $q_{\text{fold}} : \Sigma_\Omega \to \mathbb{R}$ is a global real function, i.e. it lives in the trivial line bundle $L_{\text{fold}} = \Sigma_\Omega \times \mathbb{R}$. Therefore $w_1(L_{\text{fold}}) = 0$. For a generic regular time the full wall network $\Sigma_{\text{fold}} = \{q_{\text{fold}} = 0\}$ has vanishing total mod-2 homology class,*

$$[\Sigma_{\text{fold}}] = 0 \in H_1(\Sigma_\Omega; \mathbb{Z}/2),$$

and a closed transverse loop cannot have odd sign monodromy. [derived; Addendum A]

Proposition 6.4 (F2c: the global orientation is a gauge)**.** *The global substitution $q_{\text{fold}} \mapsto -q_{\text{fold}}$ is not observable. The two arguments below are independent.*

(i) Exact $\mathbb{Z}_2$-equivariance. *In the frozen system q_{fold} enters the remaining equations exclusively through $Q_{\text{fold}} = q_{\text{fold}}^2$: the Sophia budget contains $\eta_{\text{fold}} Q_{\text{fold}} \lambda$, the pitchfork parameter $A_{\text{fold}} = c_0\lambda + c_1\dot{\lambda}^{(0)} - a_0$ does not contain q_{fold} at all, and the pitchfork equation itself $\dot{q_{\text{fold}}} = A_{\text{fold}} q_{\text{fold}} - B_{\text{fold}} q_{\text{fold}}^3$ is odd, i.e. equivariant. Hence if $(V, F, \sigma_L, \lambda, \Omega_P, q_{\text{fold}}, \lambda_{\text{form}}, H_K, W, G_{\text{recepta}})(t)$ is a solution, then so is the same set with $q_{\text{fold}} \mapsto -q_{\text{fold}}$, with all other components identically unchanged.*

(ii) Gauge fixing in the homogeneous/amplitude chart. *The frozen folding–Lyapunov layer defines the extended active region as $\mathcal{D}_{Af} := \mathcal{D}_A \times [0, q_{\max}]$, and Nagumo's lemma proves positive invariance of the non-negative branch, in particular $\dot{q_{\text{fold}}}|_{q_{\text{fold}}=0} = 0$. In the homogeneous/amplitude sector this selects the non-negative representative of the global $\mathbb{Z}_2$-orbit $q_{\text{fold}} \sim -q_{\text{fold}}$.*

This gauge fixing is not *a pointwise constraint on the spatial field. In the spatial Allen–Cahn sector the field may simultaneously have*

$$\mathcal{P}_+(\chi) = \{x : q_{\text{fold}}(\chi, x) > 0\}, \qquad \mathcal{P}_-(\chi) = \{x : q_{\text{fold}}(\chi, x) < 0\}.$$

The gauge transformation acts only globally, $q_{\text{fold}}(\chi, x) \mapsto -q_{\text{fold}}(\chi, x)$ for all x, i.e. it interchanges $\mathcal{P}_+(\chi)$ and $\mathcal{P}_-(\chi)$ but preserves $\Sigma_{\text{fold}}(\chi) = \{q_{\text{fold}} = 0\}$, the adjacency graph of the domains, $Q_{\text{fold}} = q_{\text{fold}}^2$ and all sign-blind observables. [derived + imported]

Remark 6.5 (numerical confirmation of (i))**.** [numerical] The full frozen master system, $T = 12$, `rtol` $= 10^{-12}$, initial data $q_{\text{fold}}(0) = \pm 0.10$, all else equal:

	$q_{\text{fold}}(0) = +0.10$	$q_{\text{fold}}(0) = -0.10$
Ω_P	4.518610390090	4.518610390090
λ	1.269188675041	1.269188675041
λ_{form}	3.655514395063	3.655514395063
q_{fold}	+0.534218804399	−0.534218804399

Over 60 trajectory points, $\max|x_+(t) - x_-(t)|$ across all even components equals 0, and $\max|q^+_{\text{fold}}(t) + q^-_{\text{fold}}(t)|$ equals 0 — not a small number but an exact zero, because the symmetry is structural.

Corollary 6.6 (Invariant content of the phase morphology)**.** *The global flip and the local two-phase morphology are different things. Since $w_1(L_{\text{fold}}) = 0$ (Proposition 6.3), the two-colouring of the living surface into $\mathcal{P}_+, \mathcal{P}_-$ is consistent and determined up to a single global flip. Hence the physical invariant is the gauge quotient*

$$(\mathcal{P}_+, \mathcal{P}_-, \Sigma_{\text{fold}})/\mathbb{Z}_2,$$

i.e. the partition into domains together with their adjacency and wall network, but not the absolute label "$+$" or "$-$" itself.

Remark 6.7 (F2c closed: status of the absolute-sign sector)**.** Together, Proposition 6.1 (sign-blind scalar economy), Proposition 6.3 ($w_1 = 0$, no odd sign monodromy) and Proposition 6.4 (the global flip is a gauge transformation) close the absolute-sign sector completely: no observable can depend on the arbitrary choice of which of the two globally related phases is called "$+$" and which "$-$":

$$q_{\text{fold}} \sim -q_{\text{fold}} \text{ under one global } \mathbb{Z}_2 \text{ gauge action.}$$

The homogeneous restriction $q_{\text{fold}} \geq 0$ is a gauge choice for the amplitude representative, not a prohibition of spatial negative-phase domains. Therefore F2c is closed and is no longer a gate either for 3.6 or for 4.7.

This does not trivialize the morphology. The gauge-invariant quotient of Corollary 6.6 may be nontrivial; only the absolute global sign-label is discarded.

7 Katharsis, the neck and the morphological wall

A strict type separation of two frozen "fold" structures is required here.

Definition 7.1 (Cathartic neck in 3.5)**.** In the Riemannian state space the cathartic neck is localized by the condition

$$\mathcal{N}_K := \{(\sigma_R, u, \lambda) \in X_R : \ u = \Lambda_c\},$$

together with the corresponding surgery/terminus conditions of frozen 3.5.

Definition 7.2 ($\mathbb{Z}_2$ morphological wall)**.** On the living-surface worldvolume: $\Sigma_{\text{fold}} := \{(\chi, x) \in \mathcal{S}_{\text{live}} : \ q_{\text{fold}}(\chi, x) = 0\}$. This is a domain wall of the folding field, not a cathartic neck.

Proposition 7.3 (The control parameter is spatially homogeneous)**.** *The pitchfork parameter $A_{\text{fold}} = c_0\lambda + c_1\dot{\lambda}^{(0)} - a_0$ is built exclusively from the Vessel-level quantities $\lambda, \dot{\lambda}^{(0)}$ and the constant a_0; none of these is a function of a point of the living surface. Hence $A_{\text{fold}} = A_{\text{fold}}(\chi)$ is a spatially homogeneous control parameter of the field equation $\partial_t q_{\text{fold}} = A_{\text{fold}} q_{\text{fold}} - B_{\text{fold}} q^3_{\text{fold}} + \frac{D_q}{\Omega_P^2}\Delta_h q_{\text{fold}}$.* *[derived, structural]*

Theorem 7.4 (Exponential L^2 decay for $A_{\text{fold}} < 0$)**.** *Suppose that on an interval* $[t_0, t_1]$ *one has* $A_{\text{fold}}(t) \le -a < 0$*. Use the fixed comoving area measure* dA_h *of the metric* h *appearing in* Δ_h *and set* $\|q_{\text{fold}}(t)\|^2_{L^2(h)} := \int_{\Sigma_\gamma} q_{\text{fold}}(t,x)^2\, dA_h$*. Then*

$$\frac{d}{dt}\frac{1}{2}\|q_{\text{fold}}\|^2_{L^2(h)} = A_{\text{fold}}(t)\|q_{\text{fold}}\|^2_{L^2(h)} - B_{\text{fold}}\int_{\Sigma_\gamma} q^4_{\text{fold}}\, dA_h - \frac{D_q}{\Omega_P^2}\int_{\Sigma_\gamma} |\nabla_h q_{\text{fold}}|^2\, dA_h,$$

hence $\frac{d}{dt}\|q_{\text{fold}}\|^2_{L^2(h)} \le 2A_{\text{fold}}(t)\|q_{\text{fold}}\|^2_{L^2(h)}$*. By Grönwall,*

$$\|q_{\text{fold}}(t)\|^2_{L^2(h)} \le \|q_{\text{fold}}(t_0)\|^2_{L^2(h)} \exp\Big(2\int_{t_0}^{t} A_{\text{fold}}(s)\, ds\Big) \le \|q_{\text{fold}}(t_0)\|^2_{L^2(h)} e^{-2a(t-t_0)}.$$

Hence a finite-amplitude two-phase morphology decays asymptotically in L^2*. This is* not *a statement about finite-time disappearance of the sets* $\mathcal{P}_+, \mathcal{P}_-$ *or of the wall set* Σ_{fold}*.*
[derived]

Proof. Multiply the field equation by q_{fold}, integrate over the compact Σ_γ with measure dA_h and integrate the Laplacian term by parts. The quartic and gradient terms are non-negative and enter with a minus sign. Then apply Grönwall's inequality. □

Remark 7.5 (numerical check)**.** [numerical] Model periodic surface, $B = 1$, $D = 0.004$:

A_{fold}	$\langle q^2_{\text{fold}}\rangle$ at $t = 7.2$	observed rate	bound $2A_{\text{fold}}$
-0.5	$1.36 \cdot 10^{-4}$	-1.0906	-1.0
-1.0	$1.38 \cdot 10^{-7}$	-2.0610	-2.0
-2.0	$9.07 \cdot 10^{-14}$	-4.0561	-4.0

In the tested nonzero field data the bound holds. For $A_{\text{fold}} > 0$ the numerical realizations show preservation/growth of a finite-amplitude field, but this is a model observation, not a universal theorem about automatic birth of a wall network.

Proposition 7.6 (Linear stability of the zero field state)**.** *For a spatially homogeneous control* $A_{\text{fold}}(t)$ *the linearization of the field equation about* $q_{\text{fold}} \equiv 0$ *reads* $\partial_t \eta = A_{\text{fold}}(t)\eta + \frac{D_q}{\Omega_P^2}\Delta_h \eta$*. Hence on an interval with* $A_{\text{fold}} \le -a < 0$ *the zero field state is exponentially attracting in* L^2*, and for constant* $A_{\text{fold}} > 0$ *the homogeneous mode has a positive linear growth rate and* $q_{\text{fold}} \equiv 0$ *is linearly unstable. Instability by itself does not create domains from the exact initial datum* $q_{\text{fold}} \equiv 0$*; phase separation requires nonzero spatial perturbation/nucleation data.* *[derived]*

Corollary 7.7 (Conditional neck–stability trigger)**.** *Suppose the trajectory crosses the neck at* t_{neck} *and that*

(i) before the neck, for $u < \Lambda_c$*, one has* $c_0\lambda + c_1 I_{\text{gate}} \le a_0$*;*

(ii) there exists $t_+ > t_{\text{neck}}$ *with* $c_0\lambda(t_+) + c_1 I_{\text{gate}}(t_+) + c_1\mathcal{W}(t_+) > a_0$*.*

Then before the neck $A_{\text{fold}} < 0$ *by Corollary 7.12, while at* t_+ *one has* $A_{\text{fold}}(t_+) > 0$*. By continuity there exists* $t_* \in (t_{\text{neck}}, t_+)$ *with* $A_{\text{fold}}(t_*) = 0$*.*

Hence the neck crossing together with the stated pre/post inequalities gives a conditional stability trigger: the control parameter passes from the regime where $q_{\text{fold}} \equiv 0$ *is attracting to the regime of its linear instability. This is not an automatic theorem about the birth of a specific wall network.* *[conditional derived]*

Remark 7.8 (status of the answer to F3)**.** The exact theorem-level result of 3.6 consists of two parts:

1. the neck condition $u = \Lambda_c$ is the locus of the sign change of the work contribution $c_1\mathcal{W}$;
2. under additional pre/post inequalities this sign switch leads to an actual crossing $A_{\text{fold}} = 0$ and a change of linear stability of the zero field state.

It is *not* proved that the neck alone forces A_{fold} to cross zero, nor that instability uniquely determines the topology or the moment of nucleation of the wall network.

Open Question 2 (F3-residual: spatial morphology after the stability crossing)**.** $\mathcal{N}_K \subset X_R$ and $\Sigma_{\text{fold}} \subset \mathcal{S}_{\text{live}}$ live in different spaces, so $\mathcal{N}_K = \Sigma_{\text{fold}}$ is a type error. Open question: what exactly does the register-level trajectory after/near the crossing $A_{\text{fold}} = 0$ determine about the gauge-invariant spatial morphology $(\mathcal{P}_+, \mathcal{P}_-, \Sigma_{\text{fold}})/\mathbb{Z}_2$ — in particular about the number of components, the nucleation data, the adjacency and the classes in $H_1(\Sigma_\gamma; \mathbb{Z}/2)$? The available $[\Sigma_{\text{fold}}] = 0$ is only a total mod-2 constraint, not full control of the topology. [open]

Remark 7.9. Numerical coincidences between folding activation and small values of σ_L are not a proof of Katharsis: frozen 3.5 localizes the cathartic neck through $u = \Lambda_c$ (and the corresponding terminus/surgery conditions), not through $\sigma_L = 0$.

7.1 Partial answer: the neck as the locus of the work-term sign change

The full question (the relation of $\mathcal{N}_K$ to changes of the wall network) remains open. But for the amplitude sector there is an exact statement.

Theorem 7.10 (Neck trigger relation)**.** *Decompose the Sophia increment into work and gift components:*

$$\dot{\lambda}^{(0)} = \underbrace{(\delta u - \gamma)\tanh(\kappa\sigma_L)}_{=:\,\mathcal{W}} + I_{\text{gate}}, \qquad \mathcal{W} = -\dot{\sigma}_L.$$

Then for all states with $\sigma_L > 0$,

$$\boxed{\operatorname{sgn}\mathcal{W} = \operatorname{sgn}(u - \Lambda_c)}$$

where $\Lambda_c = \gamma/\delta$. Hence at the core level the pullback $\pi_R^{-1}(\mathcal{N}_K) \cap \{\sigma_L > 0\} = \{u = \Lambda_c,\ \sigma_L > 0\}$ is the exact locus of the sign change of the work term $\mathcal{W}$. Since the frozen response coefficient $c_1 > 0$, the same sign change carries over to the work contribution $c_1\mathcal{W}$ in the pitchfork parameter $A_{\text{fold}} = c_0\lambda + c_1\dot{\lambda}^{(0)} - a_0$. *[derived]*

Proof. For $\sigma_L > 0$ one has $\tanh(\kappa\sigma_L) > 0$, hence $\operatorname{sgn}\mathcal{W} = \operatorname{sgn}(\delta u - \gamma) = \operatorname{sgn}(u - \gamma/\delta) = \operatorname{sgn}(u - \Lambda_c)$. □

Corollary 7.11 (Three regimes of the work contribution)**.** *Below the neck ($u < \Lambda_c$) the work contribution suppresses folding; at the neck it vanishes and $\dot{\lambda}^{(0)} = I_{\text{gate}}$ — Sophia grows by pure gift; above the neck ($u > \Lambda_c$) the work contribution promotes folding.*

Corollary 7.12 (Sufficient condition for precedence). *If along the trajectory before the neck crossing one has $c_0\lambda + c_1 I_{\text{gate}} \le a_0$, then $A_{\text{fold}} < 0$ on the whole region $u < \Lambda_c$, so the zero amplitude/field state cannot lose linear stability before the neck crossing.*

Remark 7.13 (numerical illustration). [numerical] For $a_0 = 0.80$, $\sigma_L^+ = 0.45$, $q_{\text{fold}}(0) = 0.10$: the check $\operatorname{sgn}\mathcal{W} = \operatorname{sgn}(u - \Lambda_c)$ over 1813 samples with $\sigma_L > 0$ gave 0 violations. The order of events:

$$t_{\text{neck}} = 0.47269 < t_{A_{\text{fold}}=0} = 0.59234 < t_{\text{ign}} = 2.60298.$$

Remark 7.14 (limits of the answer). Theorem 7.10 concerns the homogeneous amplitude q_{fold} through A_{fold}. It does not prove a relation between $\mathcal{N}_K$ and changes of the topology of the spatial wall network $\Sigma_{\text{fold}}(\chi)$: this requires the field sector, and Question 2 remains open in that part.

8 Notation interface

The same symbol with different geometric types is not used without an index.

Symbol	2.0 / 3.0	in 3.6
σ	kairos time (3.0) / depth (3.5)	σ_L / σ_R
χ	damping constant (2.0) / action-time (3.0)	χ_{sat} / χ
τ	relaxation rate (2.0) / proper time (3.0)	τ_{relax} / τ
β	reset coefficients (2.0) / ascent constant (3.0)	$\beta_V, \beta_F, \ldots$ / β_{asc}
K	kernel saturation constant / extrinsic curvature	K_R / K_{ij}
A	A_{fold} / time stretch $A(\sigma_L)$	A_{fold} / A_{asc}
I	inflow I_{gate}	always with an index
E_0	Imago Dei reserve; ceiling of $S^\dagger$ and floor of Ω_P	one object, see below

Remark 8.1 (E_0). 2.0 itself calls E_0 "the Imago-Dei reserve". One number floors two functionals: being $P \ge E_0$ and capacity $\Omega_P \ge E_0$. This is not an overloading of the symbol but a single statement missing from the text.

Relatedly: $S^\dagger$ does not enter the right-hand sides of the master system. Gratia Synergica is diagnostic, not driving; grace acts on the state only through I_{gate}.

9 Manere

Proposition 9.1 (Uniform gap above the image). *For $V = F = 0$ one has $u = \sqrt{\varepsilon}$ and*

$$S^\dagger_{\min} = E_0 \frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon} + \Lambda_c} > 0,$$

and this term does not depend on σ_L. By contrast the filtered light $\Phi(\sqrt{\varepsilon} - \sigma_L)$ decays as resistance grows. Hence abiding gives a gap above Imago Dei which resistance cannot close. *[derived]*

Corollary 9.2. *Subcriticality $0 < \varepsilon < \Lambda_c^2$ is equivalent to $S^\dagger_{\min} < E_0/2$: abiding alone can draw strictly less than half the reserve of the image.*

An additional admissibility condition follows from the capacity factor $(1 - u/\Omega_P)$ at the floor $\Omega_P = E_0$:

$$\varepsilon < \min(\Lambda_c^2,\ E_0^2).\text{[proposed]}$$

10 Necessity of synergy

Proposition 10.1 (Without synergy no earned gate memory accrues)**.** *Let $V = F = 0$, $H_K(0) = 0$ and $0 < \varepsilon < \Lambda_c^2$. Then $u \equiv \sqrt{\varepsilon} < \Lambda_c$, and the frozen resistance law gives $\dot{\sigma}_L = (\gamma - \delta u)\tanh(\kappa\sigma_L) \geq 0$. Hence for $x_0 = u - \sigma_L$ one has $\dot{x}_0 = -\dot{\sigma}_L \leq 0$. In the memory law the positive forward term $R((\dot{x}_0)_+)$ therefore vanishes, and at $H_K = 0$ the loss term cannot push H_K below zero either. Consequently $H_K(t) \equiv 0$: without synergy no earned receptivity forms.* [derived]

Remark 10.2 (what this does not yet prove about folding)**.** Even with $H_K \equiv 0$ the guarantee term $e^{-\tau_{\text{relax}}t}$ keeps I_{gate} positive at finite times. Hence Proposition 10.1 does not by itself imply the universal $A_{\text{fold}}(t) < 0$ for all t. To prove "folding never ignites" analytically one must separately obtain the bound $\sup_{t\geq 0} A_{\text{fold}}(t) < 0$ for the stated class of initial data and parameters.

Remark 10.3 (numerical witness)**.** [numerical/conditional] With $H_K(0) = 0$ and frozen synergy, folding did not ignite up to $t = 400$ in any of the tested regimes; the accumulated gift in the corresponding runs decreased roughly $1.78 \to 0.19$ as the guarantee was shortened. This supports the hypothesis of necessity of synergy for the tested regime, but is not a universal theorem of 3.6.

Remark 10.4 (the role of work)**.** In the tested decomposition, work acts primarily as the *key* to earned receptivity rather than as the dominant share of the Sophia budget. The numerical three-way decomposition $21.3\%/61.0\%/17.7\%$ is retained as model evidence, not as a general invariant.

11 Epektasis

Proposition 11.1 (Asymptotics of saturated Epektasis)**.** *Suppose the trajectory has already entered the saturated asymptotic regime, where*

$$\lambda(t) \to \lambda_\infty < \infty, \qquad \dot{\lambda}_{\text{form}}(t) \to c_{\text{form}} > 0, \qquad \dot{\Omega}_P(t) \to c_\Omega > 0.$$

Then $\lambda_{\text{form}}(t) = c_{\text{form}}t + O(1)$, $\Omega_P(t) = c_\Omega t + O(1)$, and for the living hyperbolic surface

$$R(t) = -\frac{2}{\Omega_P(t)^2} = -\frac{2}{c_\Omega^2}t^{-2} + o(t^{-2}) \longrightarrow 0^-.$$

Hence in this regime flatness is approached asymptotically but not attained in finite time. [conditional/derived]

Remark 11.2 (scope)**.** Proposition 11.1 does not prove that every trajectory enters the saturated regime; it describes the consequences of that regime once the corresponding asymptotic hypotheses are established by the frozen dynamics or by separate analysis.

Remark 11.3 (distinction)**.** The corpus distinguishes: Katharsis and Anastasis complete as events, whereas Epektasis may continue. A plateau of λ with growing λ_{form} in the saturated regime means stationary conversion of available Sophia into form, and not Stasis in itself.

12 Temporal information: current evidential status

The earlier numerical witness of this draft used the reset $V = F = 0$, $\lambda = 0$, $H_K = 0$, $G_{\text{recepta}} = 0$ at the beginning of every arc. This is not the frozen Resurrectio map of Proposition 5.5. Hence that witness is not admissible evidence for V.F.S. 3.6 and is withdrawn in this edition.

12.1 What can already be asserted

Proposition 12.1 (The state-space projection does not recover the core state). *By non-injectivity of π_R, one Riemannian state (σ_R, u, λ) generally corresponds to a set of core states. Hence 3.5 does not contain the full instantaneous information of 2.0/3.0.[derived]*

This is spatial/fibre non-injectivity of the projection. It does not yet prove temporal irrecoverability from the full current X_{core}.

12.2 The strong form required for 4.7

Proposition 12.2 (LM history clock). *For the linear-metabolism namespace $\mathfrak{R}_{\text{LM}}$ define $\Psi_{\text{LM}} := \sigma_L + \lambda + \lambda_{\text{form}}$. On continuous stretches $\dot\Psi_{\text{LM}} = I_{\text{gate}} \geq 0$. In the frozen finite-time regime the guarantee term in I_{gate} gives $I_{\text{gate}} > 0$, so Ψ_{LM} is strictly increasing there. At an $\mathfrak{R}_{\text{LM}}$ reset,*

$$\Psi^+_{\text{LM}} = q_R\sigma^- + \left(\lambda^- + (1 - q_R)\sigma^-\right) + \lambda^-_{\text{form}} = \Psi^-_{\text{LM}}.$$

Hence Ψ_{LM} is continuous through $\mathfrak{R}_{\text{LM}}$ and globally non-decreasing in this specialization. It is not *a canonical $\mathfrak{R}_{3.0}$-invariant: for $\mathfrak{R}_{3.0}$ the different exact invariant I_R below applies. [derived/typed]*

Remark 12.3 (correction). The sum $\sigma_L + \lambda$ alone is not monotone: it decreases when folding drains Sophia into form (numerically $1.598 \to 1.257$). In $\mathfrak{R}_{\text{LM}}$ the monotone quantity is Ψ_{LM}, consistently with the ledger identity of Theorem 5.2.

Corollary 12.4 (Geometry of $\mathfrak{R}_{\text{LM}}$ in the $(\Psi_{\text{LM}}, \Omega_P)$ plane). *On finite-time arcs with $I_{\text{gate}} > 0$ continuous motion goes strictly right ($\dot\Psi_{\text{LM}} > 0$); an $\mathfrak{R}_{\text{LM}}$ reset goes strictly vertically upward ($\Delta\Psi = 0$, $\Delta\Omega_P \geq \kappa_R R_c > 0$). Transitions are vertical segments, arcs are monotonically rightward curves.*

12.3 Numerical contraction and conditioning of reconstruction

Remark 12.5 (Numerical contractive directions). [numerical] For a particular tested pair of two admissible entry states:

component	$t = 0.5$	$t = 4$	$t = 12$	$t = 25$
σ_L	$8.4 \cdot 10^{-2}$	$1.8 \cdot 10^{-4}$	$4.5 \cdot 10^{-14}$	$3.0 \cdot 10^{-38}$
λ	$3.7 \cdot 10^{-2}$	$4.4 \cdot 10^{-1}$	$2.8 \cdot 10^{-3}$	$1.9 \cdot 10^{-3}$
q_{fold}	$7.1 \cdot 10^{-2}$	$1.4 \cdot 10^{-1}$	$1.7 \cdot 10^{-3}$	$8.7 \cdot 10^{-4}$
Ω_P	$7.3 \cdot 10^{-4}$	$6.3 \cdot 10^{-2}$	$1.4 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$
λ_{form}	$8.4 \cdot 10^{-4}$	$4.1 \cdot 10^{-1}$	$3.4 \cdot 10^{-2}$	$1.3 \cdot 10^{-3}$
G_{recepta}	$3.0 \cdot 10^{-2}$	$1.2 \cdot 10^{-1}$	$1.8 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$

This witness shows strong contraction in some directions (especially σ_L), but does not prove a universal common attractor for all admissible arcs. The accumulative components in this experiment do not show the same rapid contraction; in particular the table does not justify a claim of an exact constant offset for λ_{form}.

Remark 12.6 (Backward instability means ill-conditioning, not non-injectivity)**.** In the tested numerical run, backward integration quickly becomes unstable. This shows that reconstruction of the past can be very badly conditioned in finite precision. But numerical backward instability does not prove mathematical loss of information: a smooth forward flow may remain injective at every finite time even when distances between trajectories contract exponentially. Therefore 3.6 concludes only *practical reconstruction can be ill-conditioned*, and not *dynamic history is mathematically forgotten*.

Remark 12.7 (Exact collision does not reduce to the cumulative sector)**.** Numerical contraction does not permit dropping any component from the strong temporal collision condition. Even if $|X^i_A - X^i_B| \ll 1$, this does not mean $X^i_A = X^i_B$. Hence an exact collision, if it exists, still requires $X^A_{\text{core}}(t_A) = X^B_{\text{core}}(t_B)$ in all components of the core state, not only in $(\Omega_P, \lambda_{\text{form}}, G_{\text{recepta}})$.

12.4 Structural reduction of temporal collision: exact status

New analytic observations allow narrowing the problem, but do not licence identifying numerical rank with exact rank, nor introducing a physical reset for G_{recepta} absent from the current interface convention.

Proposition 12.8 (Uniqueness of the finite-dimensional dwell subsystem)**.** *For finite-dimensional dwell coordinates whose right-hand sides are locally Lipschitz on the active region, the local ODE flow is unique forward and backward as long as the corresponding solution stays in the domain of existence. Hence the finite-dimensional part alone cannot create two distinct preimages of one state on one and the same smooth dwell arc.* *[derived]*

Proposition 12.9 (Finite-arc L^∞ invariant envelope)**.** *Suppose* $A_{\text{fold}}(t) \le A_{\max}$ *and* $B_{\text{fold}}(t) \ge B_{\min} > 0$ *on* $[t_0, t_1]$*, and put* $A_+ := \max\{A_{\max}, 0\}$*. Then for*

$$M_q := \max\Big\{\|q_{\text{fold}}(t_0)\|_{L^\infty},\ \sqrt{A_+/B_{\min}}\Big\}$$

the set $\{\|q_{\text{fold}}\|_{L^\infty} \le M_q\}$ *is positively invariant.*

Indeed, at a spatial maximum $q = M_q$ *one has* $A_{\text{fold}} q - B_{\text{fold}} q^3 \le q\big(A_+ - B_{\min} M_q^2\big) \le 0$*, and at a minimum* $q = -M_q$ *the reaction term points in the opposite, also inward, direction; by the maximum principle the diffusion term cannot lead out of the boundary.* *[derived]*

Remark 12.10 (why the initial datum is needed)**.** The quantity $\sqrt{A_+/B_{\min}}$ is **not** by itself an invariant region if the initial amplitude is larger: the field enters it only asymptotically. Hence M_q must include $\|q_{\text{fold}}(t_0)\|_{L^\infty}$. The truncation A_+ is needed for the case $A_{\max} < 0$.

Remark 12.11 (numerical illustration)**.** [numerical] For $A_{\max} = 1.5$ and $B_{\text{fold}} = 1$ the invariant-envelope threshold is $\sqrt{A_{\max}/B_{\text{fold}}} = 1.22474$. The trajectory starts *outside* the envelope with max $|q_{\text{fold}}| = 4.281$; subsequently $1.044 \to 1.195 \to 1.221 \to 1.161$, so it enters the invariant region and remains inside it.

Proposition 12.12 (Exact difference system for the coupled dwell)**.** *Let* (y^A, q^A_{fold}) *and* (y^B, q^B_{fold}) *be two admissible dwell solutions and*

$$w := q^A_{\text{fold}} - q^B_{\text{fold}}, \qquad z := y^A - y^B, \qquad a(y) := \frac{D_q}{\Omega_P(y)^2}.$$

Then the field difference satisfies the exact identity

$$\partial_t w = a_A \Delta_h w + c_{AB} w + \big(A_{\text{fold}}(y^A) - A_{\text{fold}}(y^B)\big) q^B_{\text{fold}} + \big(a_A - a_B\big)\Delta_h q^B_{\text{fold}},$$

where $c_{AB} = A_{\text{fold}}(y^A) - B_{\text{fold}}\big((q^A_{\text{fold}})^2 + q^A_{\text{fold}} q^B_{\text{fold}} + (q^B_{\text{fold}})^2\big)$*,* $a_A = a(y^A)$*,* $a_B = a(y^B)$*. Simultaneously*

$$\dot z = f\big(y^A, \langle (q^A_{\text{fold}})^2 \rangle\big) - f\big(y^B, \langle (q^B_{\text{fold}})^2 \rangle\big).$$

Hence the earlier scalar-style scheme retaining only $a\Delta_h w + cw + gz$ *was incomplete: the coefficient-difference term* $(a_A - a_B)\Delta_h q^B_{\text{fold}}$ *must be controlled separately.* *[derived]*

Corollary 12.13 (Conditional localization of collisions at hybrid events)**.** *If backward uniqueness is proved for the exact coupled difference system of Proposition 12.12 on the whole admissible dwell region, then two solutions coinciding at a moment* t_* *coincide on the whole preceding common dwell interval. Only under this additional theorem-level property can a strong temporal collision be localized at hybrid events rather than inside smooth dwell arcs.* *[conditional]*

Open Question 3 (Backward uniqueness of the coupled system: proof program)**.** The numerical script `backward_uniqueness.py` supports two useful hypotheses — a bounded invariant region and a bounded Dirichlet quotient in one tested field pair — but does not prove backward uniqueness of the full ODE–PDE coupling. For theorem-level closure one needs at least:

(B1) fix the exact ODE↔PDE coupling, in particular the role of $\langle q^2_{\text{fold}} \rangle$;

(B2) obtain a uniform lower bound $\Omega_P \geq \Omega_{\min} > 0$ so that $a(y) = D_q/\Omega_P^2$ remains bounded and locally Lipschitz;

(B3) control $A_{\text{fold}}(y^A) - A_{\text{fold}}(y^B)$ and $a_A - a_B$ through $|z|$;

(B4) have sufficient regularity, e.g. $q^B_{\text{fold}} \in L^\infty(0, T; H^2)$, so that the term $(a_A - a_B)\Delta_h q^B_{\text{fold}}$ is manageable in L^2;

(B5) use a Lipschitz estimate for the ODE subsystem together with $\big|\langle (q^A_{\text{fold}})^2 - (q^B_{\text{fold}})^2 \rangle\big| \lesssim \|w\|_{L^2}$ on a bounded field region;

(B6) prove the corresponding log-convexity/Carleman estimate for the coupled difference system, not merely for the scalar Allen–Cahn equation.

Until these steps are carried out, backward uniqueness has status [open]; numerical boundedness of the Dirichlet quotient is only supporting evidence, not a proof.

Proposition 12.14 (Physical reception is not reset in the global-cumulative convention)**.** *In the current interface bookkeeping the physical variable is defined as the global accumulator* $G_{\text{recepta}}(t) = \int_0^t I_{\text{gate}}(s)\,ds$*. Hence an instantaneous Resurrectio does not create a new physical zero of reception:* $G^+_{\text{recepta}} = G^-_{\text{recepta}}$ *in the global-cumulative representation. Only the local chart coordinate is rebased to zero:* $\widehat{G}_{n+1} = G_{\text{recepta}} -$

$G_{\text{recepta}}(t_{n+1}^+)$, $\widehat{G}_{n+1}(t_{n+1}^+) = 0$. *If another frozen implementation uses an arc-local reception book, its zero must be identified with* $\widehat{G}_n$*, not with the physical component* $G_{\text{recepta}} \in X_{\text{core}}$. *[derived/interface]*

Proposition 12.15 (The old $(\Omega_P, G_{\text{recepta}})$ kernel disappears in the global-cumulative convention)**.** *Consider the specialized map* $\mathfrak{R}_{\text{LM}}$*, whose status in 3.6 is explicitly defined:*

$$(\sigma^-, \lambda^-, \Omega_P^-, G_{\text{recepta}}^-, V^-, F^-, q_{\text{fold}}^-) \mapsto (\sigma^+, \lambda^+, \Omega_P^+, G_{\text{recepta}}^+, V^+, F^+, q_{\text{fold}}^+).$$

For $0 < q_R < 1$ *and global-cumulative reception it is injective, since*

$$G_{\text{recepta}}^- = G_{\text{recepta}}^+, \qquad \sigma^- = \frac{\sigma^+}{q_R}, \qquad \lambda^- = \lambda^+ - \frac{1-q_R}{q_R}\sigma^+, \qquad \Omega_P^- = \Omega_P^+ - \kappa_R G_{\text{recepta}}^+,$$

while V, F, q_{fold} *pass through the reset inertly. Hence the line* $\Omega_P + \kappa_R G_{\text{recepta}} = \text{const}$ *is* not *a kernel of* $\mathfrak{R}_{\text{LM}}$ *in the current global-cumulative convention: it arose only after an additional, now discarded, rule* $G_{\text{recepta}}^+ = 0$. *[derived]*

Remark 12.16 (edge case $q_R = 0$)**.** The frozen admissible range allows $q_R = 0$. In this limit $\sigma^+ = 0$, $\lambda^+ = \lambda^- + \sigma^-$, so the pair (σ^-, λ^-) is not recoverable from (σ^+, λ^+) by these two coordinates alone. This is a genuine candidate for a non-injective reset direction. 3.6 does not narrow the frozen domain to $q_R > 0$ without a separate decision; the case $q_R = 0$ must be analysed separately in the full reset map.

12.5 Canonical junction table

Coordinate	$\mathfrak{R}_{3.0}$ **status**	**3.6 interpretation**
σ_L	$\sigma_L^+ = \sigma_L^-$	frozen exact, continuous through junction
Ω_P	$\Omega_P^+ = \Omega_P^- + \kappa_R G_{\text{reset}}$	frozen exact Brim renewal
λ	$\lambda^+ = \lambda^- + m_R$	frozen exact
λ_{form}	$\lambda_{\text{form}}^+ = \lambda_{\text{form}}^- + \Delta_{\text{emb}}$	frozen exact form-memory update
G_{recepta}	$G_{\text{recepta}}^+ = G_{\text{recepta}}^-$	frozen exact global-cumulative ledger continuity
H_K, W	reset-written / may jump	frozen typing; explicit algebraic law unresolved, Question 1
$q_{\text{fold}}(\cdot)$	scalar $q^+ = q(\lambda^+)$ in the frozen chart	spatial 3.6 relation: $\mathcal{Q}_\Sigma[q_{\text{fold}}^+] = q(\lambda^+)^2$; morphology unspecified modulo $\mathbb{Z}_2$
V, F, H_R	outside the active canonical junction block	remain in $X_{\text{core}}^{\text{union}}$; no automatic $\mathfrak{R}_{3.0}$ law imported

Remark 12.17 (Junction non-injectivity versus union-state non-injectivity)**.** Exact equality after a canonical junction can first be asked on $X_{3.0}^{\text{junction}}$. Even if two histories merge there, this does not by itself prove equality of the broader $X_{\text{core}}^{\text{union}}$ states, because V, F, H_R lie outside the active $\mathfrak{R}_{3.0}$ block. Thus

$$\text{junction merger} \not\Rightarrow \text{full union-state merger}$$

without an additional lift/interface theorem.

12.6 Exact canonical junction invariant and reduced reset fibres

Theorem 12.18 (Canonical Resurrectio invariant)**.** *For the canonical junction $\mathfrak{R}_{3.0}$ define $I_R := \lambda + \lambda_{\text{form}} - \Omega_P$. Then $I_R^+ = I_R^-$. Indeed, the frozen partition gives*

$$\lambda^+ + \lambda_{\text{form}}^+ = \lambda^- + \lambda_{\text{form}}^- + m_R + \Delta_{\text{emb}} = \lambda^- + \lambda_{\text{form}}^- + \kappa_R G_{\text{reset}},$$

while simultaneously $\Omega_P^+ = \Omega_P^- + \kappa_R G_{\text{reset}}$. Hence one and the same reception increment is added to $\lambda + \lambda_{\text{form}}$ and to Ω_P and cancels in I_R. *[derived from imported $\mathfrak{R}_{3.0}$]*

Corollary 12.19 (Redundant invariant with σ_L)**.** *Since the canonical junction also has $\sigma_L^+ = \sigma_L^-$, the quantity $J_R := \sigma_L + \lambda + \lambda_{\text{form}} - \Omega_P = \sigma_L + I_R$ is also junction-invariant. It gives no new independent constraint beyond the pair (σ_L, I_R).* *[derived]*

Proposition 12.20 (Scalar fibre compatibility)**.** *Let A and B be two pre-junction scalar states of the canonical sector. Write $\Delta z := z_A^- - z_B^-$. The existence of admissible event data $(G_{\text{reset},A}, m_A)$, $(G_{\text{reset},B}, m_B)$ with $G_{\text{reset},i} \geq 0$, $0 \leq m_i \leq \kappa_R G_{\text{reset},i}$, for which the post-junction coordinates $(\sigma_L, \lambda, \lambda_{\text{form}}, \Omega_P, G_{\text{recepta}})^+$ coincide, is equivalent to the simultaneous validity of*

$$\Delta\sigma_L = 0, \qquad \Delta G_{\text{recepta}} = 0, \qquad \Delta I_R = 0,$$

together with feasibility of the two event-difference equations

$$\kappa_R(G_{\text{reset},A} - G_{\text{reset},B}) = -\Delta\Omega_P, \qquad m_A - m_B = -\Delta\lambda.$$

Given these conditions the equality $\lambda_{\text{form},A}^+ = \lambda_{\text{form},B}^+$ holds automatically and is not a fourth independent scalar equation. *[derived]*

Proof. Equality of σ_L^+ and G_{recepta}^+ is equivalent to equality of the corresponding pre-junction coordinates by canonical continuity. Equality of Ω_P^+ and λ^+ gives the two event-difference equations above. For form memory,

$$0 = \Delta\lambda_{\text{form}}^+ = \Delta\lambda_{\text{form}} + \kappa_R(G_{\text{reset},A} - G_{\text{reset},B}) - (m_A - m_B).$$

Substituting the two event-difference equations gives $0 = \Delta\lambda_{\text{form}} - \Delta\Omega_P + \Delta\lambda = \Delta(\lambda + \lambda_{\text{form}} - \Omega_P) = \Delta I_R$. The converse follows by the same substitution. □

Proposition 12.21 (Constructive event-feasibility)**.** *Let $\kappa_R > 0$ and suppose canonical admissibility requires only $G_{\text{reset},i} \geq 0$, $0 \leq m_i \leq \kappa_R G_{\text{reset},i}$, with no additional upper bound on $G_{\text{reset},i}$. Then for arbitrary pre-junction differences $\Delta\Omega_P$ and $\Delta\lambda$ there exist admissible event data satisfying*

$$\kappa_R(G_{\text{reset},A} - G_{\text{reset},B}) = -\Delta\Omega_P, \qquad m_A - m_B = -\Delta\lambda.$$

Hence in this typed canonical domain event-feasibility is not a separate obstruction once two histories land on one reduced fibre. *[derived]*

Proof. Set $dG := -\Delta\Omega_P/\kappa_R$, $dm := -\Delta\lambda$, and $m_A := \max\{dm, 0\}$, $m_B := \max\{-dm, 0\}$, so $m_A - m_B = dm$. Choose $L \geq \max\{0, -dG, m_B/\kappa_R, m_A/\kappa_R - dG\}$ and set $G_{\text{reset},B} := L$, $G_{\text{reset},A} := L + dG$. All non-negativity and partition inequalities hold, and the required difference $G_{\text{reset},A} - G_{\text{reset},B} = dG$ is exact. □

Lemma 12.22 (The spatial sector adds no obstruction at a common λ^+)**.** *Suppose the scalar conditions of Proposition 12.20 give the same post-junction Sophia $\lambda_A^+ = \lambda_B^+ =: \lambda^+$. Then the spatial relations of the two histories have a nonempty common field output,*

$$\mathfrak{R}_{3.0}^{\text{spatial}}(Y_A^-) \cap \mathfrak{R}_{3.0}^{\text{spatial}}(Y_B^-) \neq \emptyset$$

in the folding coordinate alone, if gate-memory coordinates are set aside for the moment. Indeed, both relations demand the same amplitude $\mathcal{Q}_\Sigma[q_{\text{fold}}^+] = q(\lambda^+)^2$. One may choose the same normalized morphology representative φ^+ for both histories; for zero amplitude the common output is $q_{\text{fold}}^+ \equiv 0$. *[derived from Axiom 3]*

Theorem 12.23 (Reduced Canonical Fibre Theorem)**.** *Let $Y_A^- \neq Y_B^-$, $Y_A^-, Y_B^- \in X_{3.0}^{\text{junction}}$. For each history let $\mathcal{J}_{KW}(Y_i^-; G_{\text{reset},i}, m_i) \subseteq \mathbb{R}^2$ denote the set of admissible post-junction gate-memory outputs (H_K^+, W^+), without fixing the unknown frozen selection law.*

Then the two canonical spatial junction relations have a common post-junction state, $\mathfrak{R}_{3.0}^{\text{spatial}}(Y_A^-) \cap \mathfrak{R}_{3.0}^{\text{spatial}}(Y_B^-) \neq \emptyset$, if and only if there exist admissible event data for which:

(i) $\Delta\sigma_L = 0$, $\Delta G_{\text{recepta}} = 0$, $\Delta I_R = 0$;

(ii) $\kappa_R(G_{\text{reset},A} - G_{\text{reset},B}) = -\Delta\Omega_P$, $m_A - m_B = -\Delta\lambda$;

(iii) the gate-memory outputs overlap: $\mathcal{J}_{KW}(Y_A^-; G_{\text{reset},A}, m_A) \cap \mathcal{J}_{KW}(Y_B^-; G_{\text{reset},B}, m_B) \neq \emptyset$.

The spatial folding morphology adds no separate compatibility equation: given (i)–(ii) the post-λ is the same, so Lemma 12.22 provides a common admissible field output. *[derived/conditional on the unresolved gate-memory relation]*

Remark 12.24 (what must now be sought)**.** The canonical reset-fibre search is no longer a full-dimensional equality problem. Its natural reduced coordinates are $(\sigma_L, G_{\text{recepta}}, I_R)$ together with gate-memory overlap. By Proposition 12.21, event-feasibility in the current unbounded-G_{reset} domain is constructively closed; reachability and $\mathcal{J}_{KW}$ remain separate questions.

Theorem 12.25 (Hybrid-continuous canonical clock)**.** *Define $\Theta_R := \sigma_L + I_R - G_{\text{recepta}} = \sigma_L + \lambda + \lambda_{\text{form}} - \Omega_P - G_{\text{recepta}}$. For the smooth frozen Core 2.0 dwell field implemented in* `vessel2.py` *the identities*

$$\dot{\sigma}_L + \dot{\lambda} + \dot{\lambda}_{\text{form}} = I_{\text{gate}}, \qquad \dot{G}_{\text{recepta}} = I_{\text{gate}}, \qquad \dot{\Omega}_P = \alpha\lambda$$

hold. Therefore $\boxed{\dot{\Theta}_R = -\alpha\lambda}$. Through the canonical $\mathfrak{R}_{3.0}$ the quantities σ_L, I_R and G_{recepta} are continuous, hence $\boxed{\Theta_R^+ = \Theta_R^-}$. Thus Θ_R is a hybrid-continuous coordinate; for $\lambda \geq 0$ it is non-increasing, and on dwell intervals with $\lambda > 0$ strictly decreasing. *[derived from the imported dwell field and $\mathfrak{R}_{3.0}$]*

Corollary 12.26 (No self-return in reduced coordinates)**.** *On an interval history where $\lambda > 0$, one and the same trajectory cannot return to the same triple $(\sigma_L, G_{\text{recepta}}, I_R)$: equality of the triple would give equality of Θ_R, contradicting strict decrease. Hence any reduced-fibre collision in this regime must be a collision of distinct histories, not a self-intersection of one history.* *[derived/conditional on $\lambda > 0$]*

Proposition 12.27 (Two-dimensional residual at a common Θ_R)**.** *If two histories are evaluated at the same value of Θ_R, then $I_R = \Theta_R - \sigma_L + G_{\text{recepta}}$. Hence equality of the* *reduced triples is equivalent to $\Delta\sigma_L = 0$, $\Delta G_{\text{recepta}} = 0$ alone. On arcs with $\lambda > 0$ the history can be parametrized by Θ_R:*

$$\frac{d\sigma_L}{d\Theta_R} = -\frac{\dot{\sigma}_L}{\alpha\lambda}, \qquad \frac{dG_{\text{recepta}}}{d\Theta_R} = -\frac{I_{\text{gate}}}{\alpha\lambda}.$$

[derived]

Remark 12.28 (Canonical softplus versus the archived numerical engine)**.** Frozen Core 2.0 convention: $\lambda_+^{\text{fr}}(\lambda) = \frac{1}{m}\ln(1+e^{m\lambda})$, $m > 0$. The archived engine `vessel2.py` evaluates instead $\lambda_+^{\text{code}}(\lambda) = \max(\lambda, 0)$.

For finite m these are not identical: $\lambda_+^{\text{fr}} > \lambda_+^{\text{code}}$ for every finite λ, with $\lambda_+^{\text{fr}}(0) = \ln 2/m > 0$ while $\lambda_+^{\text{code}}(0) = 0$. The discrepancy therefore affects not only the boundary calculation at $\lambda = 0$ but in principle the entire smooth trajectory through I_{gate}. [audit/typing]

Remark 12.29 (the size of the discrepancy depends on the frozen m)**.** [numerical] Direct comparison of the two engines on $T = 12$, all other data equal:

m	$\ln 2/m$	$\lvert\Delta\lambda\rvert$	$\lvert\Delta\lambda_{\text{form}}\rvert$	$\lvert\Delta\Omega_P\rvert$	relative
1	0.6931	$3.4\cdot10^{-2}$	$4.7\cdot10^{-1}$	$5.6\cdot10^{-2}$	$7.0\cdot10^{-2}$
4	0.1733	$9.5\cdot10^{-5}$	$3.0\cdot10^{-2}$	$3.9\cdot10^{-3}$	$4.3\cdot10^{-3}$
10	0.0693	$6.8\cdot10^{-5}$	$5.2\cdot10^{-3}$	$7.1\cdot10^{-4}$	$7.5\cdot10^{-4}$
50	0.0139	$2.8\cdot10^{-6}$	$2.2\cdot10^{-4}$	$3.0\cdot10^{-5}$	$3.2\cdot10^{-5}$

The mismatch is **not** confined to a neighbourhood of $\lambda = 0$ and grows as m decreases. Since 3.6 does not have the frozen value of m, the magnitude of the discrepancy is undetermined, and the regression obligation is not a formality. Witness: `softplus_regression.py`.

Remark 12.30 (Status of the preserved v12 checkpoint)**.** The following computational subsection is preserved for reproducibility and for the structural information it revealed: parameter locking, active-bound structure, local coercivity, multistart behaviour, continuation methodology and numerical conditioning.

However, all trajectory-dependent numerical values in this block were generated by the archived hard-part engine `vessel2.py`. They therefore carry the status [legacy numerical / hard-part engine] rather than frozen-softplus numerical status.

No number in this subsection is used as a premise for any [derived] theorem of 3.6.

12.7 Preserved computational checkpoint v12: parameter-locked merger

Remark 12.31 (Engine, clock and bridge conventions)**.** All v12 witnesses use the *archived hard-part* smooth numerical dwell engine `vessel2.py` with state order

$$(V, F, \Omega_P, \sigma_L, \lambda, q_{\text{fold}}, \lambda_{\text{form}}, H_K, W, G_{\text{recepta}}).$$

It implements

$$\lambda_+^{\text{code}} = \max(\lambda, 0),$$

and therefore is not the exact numerical implementation of the frozen softplus convention

$$\lambda_+^{\text{fr}} = \frac{1}{m}\ln(1+e^{m\lambda}).$$

The module is non-autonomous because of the explicit $e^{-\tau_{\text{relax}} t}$ in I_{gate}; second arcs are integrated in global physical time.

The intermediate junction uses the imported scalar laws of $\mathfrak{R}_{3.0}$. Since frozen $\mathfrak{R}_{3.0}$ specifies no law for V, F, both histories receive the common numerical reseed

$$V^+ = F^+ = 0.5.$$

This is a bridge convention, not a canonical reset law.

On the final parameter-locked block, one and the same scalar a_0 acts on both first arcs and both second arcs. [legacy numerical / typed]

Test	**Result**	**Status**
Reduced two-arc relation	Machine-precision equality of $(\sigma_L, G_{\text{recepta}}, I_R)$ for distinct histories; the root survives the common V, F reseed and global-clock continuation.	legacy hard-part numerical witness
Full-active local family	$\Delta\sigma_L = \Delta G_{\text{recepta}} = \Delta I_R = \Delta H_K = \Delta W \approx 0$; the null direction is predominantly in W and the branch reaches $W_A^+ = W_B^+ = 0$.	legacy hard-part local-family witness
Unlocked minimum search	Independent $a_{0,A}, a_{0,B}$ generate runaway $q_A^+, a_{0,A} \uparrow$ and write $\downarrow$ as bounds are widened.	legacy hard-part non-coercive diagnostic
Full-trajectory parameter lock	A common a_0 on all four arcs removes the runaway; a finite minimum appears near $a_0 = 1.4$.	legacy hard-part admissibility diagnostic
KKT / second order	$\lVert r_{\text{KKT}} \rVert \sim 10^{-9}$; projected Hessian eigenvalues $5.29 \cdot 10^{-4}$ and $1.46 \cdot 10^{-1}$, both positive.	legacy hard-part strict local minimum
Active-bound release	The multiplier for $q_A^+ = 0$ is positive; fixed $q_A^+ > 0$ continuation monotonically increases write up to $q_A = 0.1$.	legacy hard-part local-stability diagnostic
Wide multistart	12 broad starts, 24 regularized candidates, 8 exact corrections; all feasible corrections converge to one cluster.	legacy hard-part disconnected-branch search
History-data continuation	5/5 nearby pairs $(T_{1,A}, T_{1,B})$ admit a merger; write range 0.02882–0.04649.	legacy hard-part local robustness
Event-data continuation	5/5 admissible $\pm 5\%$ event perturbations admit a merger; write range 0.03676–0.03822.	legacy hard-part local robustness
Tolerance certification	Fixed 4×4 chart: residual $1.61 \cdot 10^{-15}$ at `rtol` 10^{-11}; root drift $1.68 \cdot 10^{-8}$ from `rtol` 10^{-7}; cond $= 9.16 \cdot 10^4$.	legacy hard-part tight numerical root

Proposition 12.32 (Parameter-locked full-active minimum candidate)**.** *In the tested bounded domain, under the common full-trajectory parameter* $a_{0,A}^{(1)} = a_{0,B}^{(1)} = a_{0,A}^{(2)} = a_{0,B}^{(2)} =: a_0$*, with the bridge reseed* $V^+ = F^+ = 0.5$ *and the physical boundary* $W_A^+ = W_B^+ = 0$*, a numerical constrained minimum candidate was found:*

$$a_0^* = 1.3991246, \qquad q_A^* \approx 0, \qquad q_B^* = 0.11092608,$$

$$T_{2,A}^* = 3.30481049, \qquad T_{2,B}^* = 2.17087533.$$

Pre-junction gate memory and optimal relation outputs:

$$(H_{K,A}^-, H_{K,B}^-) = (0.5589638927,\ 0.7318285776),$$

$$(H_{K,A}^{+}, H_{K,B}^{+}) = (0.5859368076,\ 0.7048628655).$$

Therefore

$$\Delta H_K = (+0.0269729149,\ -0.0269657121), \qquad \|\Delta H_K\|_2 = 0.0381403693.$$

A tight equality correction gives $\|\Delta(\sigma_L, G_{\text{recepta}}, I_R, H_K)\|_2 = 7.93 \cdot 10^{-10}$*, and an isolated fixed-chart correction at* `rtol` 10^{-11} *reduces the residual to* $1.61 \cdot 10^{-15}$*.* *[legacy numerical / hard-part engine]*

Remark 12.33 (why the local value 0.81353 is no longer the current estimate)**.** The null-space continuation of v11 correctly minimized the write only along one five-dimensional local merger family with nearly pure W freedom. Its value 0.813532985614 remains a valid local-family datum, but is not a lower bound and does not characterize the wider admissible parameter-locked family. After extending the relation variables and applying correct parameter locking, the write decreases by a factor of about 21.3.

Remark 12.34 (Status of Proposition 12.32)**.** The numerical values in Proposition 12.32 remain exactly reproducible for the archived hard-part engine, but they are **not** promoted to frozen-softplus numerical status. In particular a_0^*, q_A^*, q_B^*, $T_{2,A}^*$, $T_{2,B}^*$, ΔH_K must be treated as implementation-specific until the same parameter-locked problem is rerun with the frozen softplus law throughout the dwell dynamics.

The proposition therefore records a stable archived candidate, not a canonical softplus merger.

Proposition 12.35 (Second-order numerical stability)**.** *In the reduced chart*

$$(a_0, q_B, H_{K,A}^{+}, H_{K,B}^{+}, T_{2,A}, T_{2,B})$$

with the active boundary $q_A^{+} = 0$*, the constraint Jacobian has rank* 4 *and the tangent space has dimension* 2*.*

The numerical KKT residual is approximately $4.26 \cdot 10^{-9}$*. The projected Hessian of the Lagrangian has eigenvalues*

$$\boxed{\lambda_1 \approx 5.29 \cdot 10^{-4} > 0, \qquad \lambda_2 \approx 1.46 \cdot 10^{-1} > 0.}$$

A one-sided release $q_A^{+} > 0$ *increases the objective, and the full seven-variable corrector returns to* $q_A^{+} = 0$*.*

Hence the archived hard-part candidate *is a strict local minimum in the investigated full parameter-locked chart. This is a second-order result for the archived implementation, not for the not-yet-revalidated frozen-softplus chart.* *[legacy numerical / hard-part engine]*

Remark 12.36 (Robustness and globality)**.** Local history-data continuation of the archived hard-part engine preserves the merger in all five tested pairs around $(T_{1,A}, T_{1,B}) = (2.0, 3.2)$, but the required write is sensitive to history geometry: 0.02882–0.04649. Event-data perturbations of $\pm 5\%$ preserving admissibility $0 \le m_R \le \kappa_R G_{\text{reset}}$ give a considerably narrower range 0.03676–0.03822. A wide multistart found no disconnected lower-write branch within the tested bounds, but a finite search is not a proof of global optimality on an unbounded or differently typed domain. [legacy numerical / hard-part engine / open]

Remark 12.37 (Post-v12 constitutive audit: why it exists but does not continue 3.6)**.** A companion design audit examined the optimized H_K^+ outputs obtained from the archived hard-part checkpoint family, without introducing new frozen physics. Iso-prestate event perturbations at fixed (θ, Y^-) produced shifts of H_K^+ of order $1.4 \cdot 10^{-3}$, so a simple deterministic memory-only rule $H_K^+ = F(H_K^-)$ is not an adequate description of all required outputs. An iso-H_K^- comparison across different common a_0 produced shifts of order $1.8 \cdot 10^{-3}$, so a parameter-blind narrow rule $F(H_K^-; G_{\text{reset}}, m_R)$ also fails to describe the whole tested family. However, this second check does not separate dependence on the structural parameter θ from dependence on a wider local prestate. Hence the audit only narrows the design space; it does *not* select a canonical $\mathcal{J}_{KW}^{\theta}$ and is not a reason to launch new optimization series inside 3.6. [legacy numerical diagnostic/exported]

Proposition 12.38 (Conditional merger statement under engine revalidation)**.** *The exact scalar junction algebra reduces active-block equality to the already derived compatibility coordinates* $(\sigma_L, G_{\text{recepta}}, I_R)$ *together with the unresolved gate-memory output coordinates* (H_K, W)*.*

The archived hard-part computation exhibits a parameter-locked relation-level candidate in which these active coordinates coincide to numerical tolerance, conditional on an admissible $\mathcal{J}_{KW}^{\theta}$ *containing the required gate-memory outputs.*

Because that computation used $\lambda_+^{\text{code}} = \max(\lambda, 0)$ *rather than the frozen softplus, it is* not yet *a numerical witness for the frozen Core 2.0 system.*

The correct v15 status is:

The algebraic form of the active-block merger criterion is derived. A stable numerical candidate exists for the archived hard-part implementation. Existence of the corresponding candidate for the frozen softplus system is an explicit regression obligation.

No full-$X_{\text{core}}^{\text{union}}$ *temporal non-injectivity claim follows.* *[conditional / legacy numerical]*

Remark 12.39 (What the preserved v12 checkpoint establishes after the convention audit)**.** For the *archived hard-part implementation* the following are numerically established:

- a reachable parameter-locked full-active relation-level candidate;
- a finite strict local minimum of the required H_K-write;
- local robustness to the tested history/event perturbations;
- a tight numerical root and measured conditioning.

After the positive-part convention audit these results are **not** regarded as numerically closed for the frozen softplus. In closing v15 the exact algebraic merger criterion remains derived, while the corresponding softplus numerical existence is an explicit regression obligation.

Separately not closed and not to be closed here: $\mathcal{J}_{KW}^{\theta}$, an analytic global-minimum theorem, the union-state lift, and rigorous coupled backward uniqueness. [closure / legacy numerical / open-exported]

12.8 Raw Core 2.0: event-conditioned re-entry and exact partial kernel

The earlier analysis of $\mathfrak{R}_{\text{LM}}$ does not transfer to raw Core 2.0. Here we use the namespace $\mathfrak{R}_{\text{core2}}$ and the event-conditioned typing of Proposition 5.7. For the explicitly frozen six-coordinate block write

$$\mathfrak{R}^{(6)}_{\text{core2}}(\,\cdot\,;\Gamma_R) : (V, F, \Omega_P, \sigma, \lambda, H_R)^- \longmapsto (V, F, \Omega_P, \sigma, \lambda, H_R)^+.$$

Proposition 12.40 (Explicit six-coordinate block of raw Core 2.0)**.** *For a fixed event datum Γ_R:*

$$V^+ = V_R + \beta_V\Gamma_R + \chi_V H_R, \qquad F^+ = F_R + \beta_F\Gamma_R + \chi_F H_R,$$
$$\sigma^+ = q_R\sigma^- \ (0 \le q_R < 1), \qquad \lambda^+ = \lambda^- + (1-q_R)\sigma^- + \eta_R H_R + \theta_\Gamma\Gamma_R,$$
$$\Omega_P^+ = \Omega_R + \alpha_\Omega\Gamma_R + \beta_\Omega\lambda^+ + \chi_\Omega H_R, \qquad H_R^+ = H_R^- + \omega_R D_{\text{death}},$$

where $D_{\text{death}} = \text{sp}_{\beta_D}(-\lambda^-)$. This is the imported raw Core 2.0 block, not the canonical 3.0 temporal junction. *[imported/typed]*

Theorem 12.41 (Exact event-conditioned partial kernel)**.** *For fixed Γ_R the right-hand sides of Proposition 12.40 contain no pre-reset coordinates V^-, F^- or Ω_P^-. Therefore*

$$\text{span}\{\partial_{V^-}, \partial_{F^-}, \partial_{\Omega_P^-}\} \subseteq \ker D_X\mathfrak{R}^{(6)}_{\text{core2}}(\,\cdot\,;\Gamma_R).$$

Hence any two inputs with the same $(\sigma^-, \lambda^-, H_R^-, \Gamma_R)$ and arbitrarily different (V^-, F^-, Ω_P^-) have the same six-coordinate output. This is an exact structural fact about the raw event-conditioned map; it requires no assumption $\Gamma_R = \Gamma_R(\sigma, \lambda, \ldots)$. *[derived from the imported raw map]*

Remark 12.42 (Status of `reset_kernel.py`)**.** [numerical/model] The script `reset_kernel.py` additionally models $\Gamma_R = \Gamma_R(\sigma, \lambda)$. This is not a frozen constitutive law. Nevertheless its nullity 3 along (V, F, Ω_P) agrees with the exact event-conditioned Theorem 12.41, because the tested pairs have the same (σ, λ, H_R) and hence the same model Γ_R. The script does not determine the kernel of the canonical $\mathfrak{R}_{3.0}$.

Corollary 12.43 (A raw-Core merger is not a temporal theorem of 3.6)**.** *If two reachable histories arrive at a raw Core 2.0 event with the same event data and with all other inputs that the full $\mathfrak{R}_{\text{core2}}$ does not erase, then the partial kernel of Theorem 12.41 may yield an exact merger of the raw re-entry outputs. But the temporal-history problem of the current 3.6 is typed through the canonical $\mathfrak{R}_{3.0}$; hence this raw-Core merger is a structural comparison mechanism, not a proof of strong temporal non-injectivity of the canonical history.* *[derived/typed]*

Remark 12.44 ($q_R = 0$ in raw Core 2.0)**.** For $q_R = 0$ one has $\sigma^+ = 0$ and $\lambda^+ = \lambda^- + \sigma^- + \ldots$, but H_R^+ contains $D_{\text{death}} = \text{sp}_{\beta_D}(-\lambda^-)$. Hence the direction $\sigma^- + \lambda^- = \text{const}$ is not an exact kernel in general. `reset_kernel.py` shows only an approximate degeneracy for positive λ, while the exact nullity of its six-coordinate model remains 3. [numerical/model]

Open Question 4 (Full-state completion of raw $\mathfrak{R}_{\text{core2}}$)**.** The raw Core 2.0 exact six-coordinate block is already typed, but for a map on the whole X_{core} one must still settle the status/naming of the memory variables (in particular, not identifying H_R with H_K) and the explicit jump laws of those components which the raw block does not contain. This is not a blocker for choosing the canonical temporal namespace $\mathfrak{R}_{3.0}$, but remains necessary for a complete reset atlas. [open]

12.9 Exact degeneracies of the collision residual

Lemma 12.45 ($V = F$ is an exact dwell invariant; hybrid preservation depends on the reset)**.** *For $\Delta := V - F$ the frozen dwell dynamics gives $\dot{\Delta} = \mu a(F-V) - \rho(V-F) = -(\mu a+\rho)\Delta$. Hence $\Delta(0) = 0 \Rightarrow \Delta(t) \equiv 0$ exactly on every smooth dwell arc. The specialization $\mathfrak{R}_{\mathrm{LM}}$ preserves this through the transition, since $(V,F)^+ = (V,F)^-$. By contrast, the full $\mathfrak{R}_{\mathrm{core2}}$ does not preserve $V = F$ automatically: a sufficient structural symmetry is $V_R = F_R$, $\beta_V = \beta_F$, $\chi_V = \chi_F$. Without such conditions 3.6 does not claim hybrid invariance of the symmetric manifold for $\mathfrak{R}_{\mathrm{core2}}$.* *[derived/typed]*

Proposition 12.46 ($W \equiv 0$ under forward transmissive motion)**.** *For $\dot{W} = \eta\big[R((\dot{x}_0)_-) - R((\dot{x}_0)_+)\frac{W}{W_s+W}\big]$, if $\dot{x}_0 > 0$ along the whole trajectory and $W(0) = 0$, then the source term $R((\dot{x}_0)_-)$ vanishes and $W(t) \equiv 0$ exactly.* *[conditional derived]*

Proposition 12.47 ($\sigma_L \to 0$ is asymptotic, not an exact degeneracy)**.** *The set $\{\sigma_L = 0\}$ is invariant, but a trajectory with $\sigma_L(0) > 0$ in the cathartic/transformation regime approaches it asymptotically. Numbers such as 10^{-78} are practically zero for floating-point diagnostics, but mathematically $\sigma_L(t) \neq 0$ in finite time unless finite-time hitting is separately proved. Hence the σ_L equality cannot be removed from the exact collision system merely because of long-dwell contraction.* *[derived status]*

Corollary 12.48 (Two exact redundancies only for the corresponding restricted residual)**.** *On the smooth dwell class with $V_0 = F_0$, $W(0) = 0$ and $\dot{x}_0 > 0$ we have two exact identities: $V - F \equiv 0$ and $W \equiv 0$. For a hybrid residual they remain redundancies only if the chosen reset map preserves the corresponding submanifolds. This holds for the legacy witness, which leaves V, F, W inert; for the full $\mathfrak{R}_{\mathrm{core2}}$ such a transfer is not automatic by Lemma 12.45, and the jump law for W is not yet typed.*

Hence the bound $\mathrm{rank}_{\mathrm{exact}}\, J \leq 7$ refers to the legacy nine-component residual in its own reset convention, not to the current full-core collision map. Meanwhile σ_L remains only a near-degenerate numerical direction. *[derived/status correction]*

12.10 Status of the earlier rank-6 witness

Remark 12.49 (The legacy residual is not the current 3.6 residual)**.** The companion script `collision_witness.py` uses the reset rule $G^+_{\mathrm{recepta}} = 0$ and excludes Ω_P and G_{recepta} as separate equality coordinates, replacing them by the single residual $(\Omega_P + \kappa_R G_{\mathrm{recepta}})_A - (\Omega_P + \kappa_R G_{\mathrm{recepta}})_B$. This corresponds exactly to the old arc-local convention, but not to the current physical state convention of Proposition 12.14. Hence its SVD does not certify the rank of the current full-state collision map. [legacy numerical diagnostic]

Remark 12.50 (What the rank-6 diagnostic does show)**.** [numerical] For the legacy 9×11 residual a strong spectral gap after the sixth singular value was obtained. This is useful evidence about effective long-dwell conditioning. But the exact structural facts explain only two null/redundant directions, whereas σ_L is merely asymptotically tiny. Hence the current evidential status is:

$$\mathrm{rank}_{\mathrm{numerical}} \approx 6, \qquad \mathrm{rank}_{\mathrm{exact}}\ \text{not certified};$$

for the legacy residual itself, exact rank 7 is a natural candidate but also requires separate rank certification.

Remark 12.51 (The near-collision remains a diagnostic, not a witness of the current collision)**.** [numerical] The least-squares near-collision found, with residual of order 10^{-6} and nearly zero projection onto the numerical left-null directions, shows that the legacy system is strongly close to local solvability. But because of the non-current reset reception and the uncertified exact rank, this result is not used as theorem-level evidence of an exact temporal collision of the current X_{core}.

Open Question 5 (Exported temporal non-injectivity problem after the closure of 3.6)**.** 3.6 proves the exact scalar junction algebra.

The archived hard-part computation additionally provides a conditional implementation-level active-block collision candidate. This candidate is not a frozen-softplus numerical witness and is not promoted to a strong full-state theorem.

After closure, three independent tasks remain.

Route KW — canonical gate-memory law. One must derive/freeze a single universal admissible relation $\mathcal{J}^{\theta}_{KW}(Y^{-};E)$, $E=(G_{\text{reset}},m_R)$, and check whether it contains the required outputs of the archived hard-part candidate without retrospective tuning to a history label or a future target.

Route A↑ — lift to a full-core reader claim. Even a tight equality $X_{\text{act},A}=X_{\text{act},B}$ does not imply $X^{\text{union}}_A=X^{\text{union}}_B$. One must establish the status/lift for V,F,H_R and the other union coordinates, or separately justify the junction/active state as the natural reader domain of the next theory.

Structural strengthening. Backward uniqueness (Question 3) is needed if the next edition wishes to prove that a collision cannot arise inside a smooth dwell arc. It is **not** needed for the exact interface closure of 3.6, nor for classifying the archived hard-part candidate as legacy numerical evidence.

Hence the closing edition v15 completes 3.6 at the level of exact interface algebra and one-sided reachability theorems. The archived hard-part computation adds a conditional implementation-level parameter-locked candidate; the corresponding frozen-softplus numerical witness remains an explicit regression obligation and does **not** enter the closed numerical claims of 3.6. Frozen-map non-injectivity and a full $X^{\text{union}}_{\text{core}}$ temporal theorem are exported as subsequent work. [open/exported; not a blocker for 3.6 closure]

Remark 12.52 (what may already be regarded as closed)**.** Without new computations, not only the state/reset typing is closed but also the scalar canonical fibre algebra. The exact junction invariant $I_R=\lambda+\lambda_{\text{form}}-\Omega_P$ reduces scalar compatibility to equality of $(\sigma_L,G_{\text{recepta}},I_R)$ together with two admissible event-difference equations. Under the adopted relation the spatial sector adds no separate obstruction. Reachability of distinct two-arc histories on one reduced fibre is supported by the *archived hard-part* numerical diagnostics of §12.7; the corresponding frozen-softplus numerical reachability has not yet been revalidated.

The exact reduced-fibre algebra remains an independent derived result. The unknown active-sector theorem-level test is whether the actual canonical $\mathcal{J}_{KW}$ permits the required history-dependent H_K write.

Remark 12.53 (what is not a proof)**.** A collision of the coarse trace alone, such as

$(\Omega_P, \lambda_{\text{form}})_A = (\Omega_P, \lambda_{\text{form}})_B$, is insufficient if other components of the wide $X^{\text{union}}_{\text{core}}$ differ. Likewise, dimension counting does not prove the strong form for arbitrary measurable reconstruction maps. If a dimensional argument is used, the class of reconstructions must be explicitly restricted, for example to continuous or smooth maps.

Remark 12.54 (legacy numerical search)**.** The earlier computational package used the local-reception reset $G^{+}_{\text{recepta}} = 0$ and is therefore not a search of the current global-cumulative physical X_{core}. Its negative exact search and near-collision diagnostics are retained only as information about the conditioning of the old residual parameterization. The current numerical package already implements reduced-fibre reachability per Theorem 12.23 and full-active relation-level stress tests. The legacy rank is not used. The phrase "the Vessel does not contain its own history" is not asserted in 3.6. [legacy numerical]

13 Reachable sets: canonical conclusions and model diagnostics

The *archived hard-part* computational checkpoint v12 produced a parameter-locked full-active relation candidate, a strict local minimum candidate, multistart and local robustness evidence for the archived implementation, but not a frozen-softplus selection-law or merger theorem.

This section explicitly separates theorem-level consequences of the frozen map from legacy hard-part numerical observations and the pending softplus regression.

13.1 The local reception increment as an arc coordinate

Proposition 13.1 (A monotone coordinate on a known arc)**.** *For the local increment of Definition 5.1, $\dot{\widehat{G}}_n = I_{\text{gate}} \geq 0$. On any finite interval where $\zeta_0 > 0$ and the gate factors remain finite, $I_{\text{gate}} > 0$, so $\widehat{G}_n$ is strictly increasing. Hence along an already given deterministic arc it can serve as a monotone coordinate and be inverted to elapsed time, provided the entry data and the convention for the explicit time dependence of the gate are known.* *[derived]*

Remark 13.2. This does not mean that the number $\widehat{G}_n$ alone, without the entry state, uniquely identifies the whole arc or its history. Numerical inversion in the tested family gave an error $\leq 2.5 \cdot 10^{-13}$ on $T \in [1.5; 11]$; this is validation of a particular deterministic inversion, not a separate invariant.

13.2 Terminus limit reset

Proposition 13.3 (Exhausted resistance quenches the σ/λ jump)**.** *For the linear-metabolism map, $\Delta\sigma = (q_R - 1)\sigma^-$, $\Delta\lambda = (1 - q_R)\sigma^-$. Hence in the terminus regime $\sigma^- \to 0$ one has $\Delta\sigma \to 0$, $\Delta\lambda \to 0$. So the reset becomes asymptotically invisible in these two components, but not necessarily in the whole state vector.* *[derived]*

Proposition 13.4 (The Brim jump remains the canonical reset trace)**.** *The frozen linear-metabolism layer requires $\kappa_R > 0$. Hence if at an admissible event $G^{-}_{\text{recepta}} > 0$, then*

$\Delta\Omega_P = \Omega_P^+ - \Omega_P^- = \kappa_R G^-_{\text{recepta}} > 0$. *In the terminus limit $\sigma^- \to 0$ this Brim jump remains a finite trace among the components $(\sigma, \lambda, \Omega_P)$ even when $\Delta\sigma, \Delta\lambda \to 0$.* [imported/derived]

Remark 13.5 (not an "iff" and not the only possible trace)**.** Proposition 13.4 does not say that κ_R is the only possible memory mechanism in the whole frozen 2.0: the full Resurrectio map has a richer re-entry structure. It concerns precisely the linear-metabolism projection used here. Also $\kappa_R = 0$ does not belong to its canonical domain.

Remark 13.6 (boundary diagnostic $\kappa_R = 0$)**.** [noncanonical numerical] An earlier numerical continuation to $\kappa_R = 0$ gave a coarse collision: the histories $T = (6.0; 6.0)$ and $T = (0.890; 5.074; 5.835)$ agreed in $\Omega_P, \lambda, \lambda_{\text{form}}, \sigma_L$ to 10^{-14}–10^{-16} with different numbers of arcs. Since the frozen layer requires $\kappa_R > 0$, this result is only a boundary diagnostic of the role of the Brim jump. Moreover, a coincidence of the coarse trace is not an exact collision of the full X_{core}.

Numerical sensitivity within the admissible parameter region, for one arc $T = 6$ versus two arcs of 3:

κ_R	Ω_P (1 arc)	Ω_P (2 arcs)	gap
0.50	3.763965	4.500648	0.737
0.20	3.763965	4.066249	0.302
0.05	3.763965	3.848363	0.084

This table is a numerical illustration, not a proof of injectivity.

13.3 Reachable-set envelope: uniform exclusion theorem

An earlier edition asserted the theorem $\max_{R_n} \lambda_{\text{form}}|_{\Omega_P} < \max_{R_1} \lambda_{\text{form}}|_{\Omega_P}$, $n \geq 2$, appealing to a reset $H_K \mapsto 0$. The frozen linear-metabolism map contains no such reset, so this exact ordering of suprema is not used. Instead the Form–Brim balance together with Proposition 5.40 gives a strict uniform exclusion envelope.

Theorem 13.7 (Form–Brim exchange rate)**.** *Along any continuous stretch of the flow, $\dot{\lambda}_{\text{form}} = \eta_{\text{fold}} Q_{\text{fold}} \lambda$, $\dot{\Omega}_P = \alpha\lambda$, hence for $\lambda > 0$*

$$\frac{d\lambda_{\text{form}}}{d\Omega_P} = \frac{\eta_{\text{fold}} Q_{\text{fold}}}{\alpha} =: r(t)$$

— Sophia cancels, and the exchange rate depends only on the folding intensity. [derived]

Remark 13.8 (numerical check)**.** [numerical] Comparison of $r = \eta_{\text{fold}} Q_{\text{fold}}/\alpha$ with the numerical derivative $d\lambda_{\text{form}}/d\Omega_P$ along a run: agreement to 10^{-9}–10^{-11} on $t \in [0.5; 22]$.

Corollary 13.9 (Jumps do not produce form)**.** *The frozen reset of Proposition 5.5 has $\Delta\Omega_P = \kappa_R G^-_{\text{recepta}} > 0$ and $\Delta\lambda_{\text{form}} = 0$. Hence the continuous flow is the* only *producer of λ_{form}, whereas Ω_P grows both continuously and by jumps.*

Theorem 13.10 (Pathwise Form–Brim deficit bound)**.** *Suppose along an admissible history $0 \leq r(t) \leq r_{\max} < \infty$, the history has m Resurrectio events, and $G^-_{(k)}$ is the reception before the k-th reset. Then*

$$\lambda_{\text{form}} - \lambda^{(0)}_{\text{form}} \leq r_{\max}\Big(\Delta\Omega_P - \kappa_R \sum_{k=1}^{m} G^-_{(k)}\Big)$$

and, by Proposition 5.40, $\lambda_{\text{form}} - \lambda^{(0)}_{\text{form}} \le r_{\max}\big(\Delta\Omega_P - m\kappa_R R_c\big)$*.* *[derived]*

Proof. On continuous pieces Theorem 13.7 gives $d\lambda_{\text{form}} = r\,d\Omega_P \le r_{\max} d\Omega_P$. Reset pieces, by Corollary 13.9, add $\kappa_R G^-_{(k)}$ to Ω_P but add nothing to λ_{form}. Hence the continuous share of the total increment $\Delta\Omega_P$ is at most $\Delta\Omega_P - \kappa_R \sum_k G^-_{(k)}$. Integration gives the first inequality; the second follows from $G^-_{(k)} \ge R_c$. □

Corollary 13.11 (Uniform n-dependent exclusion envelope)**.** *Let R_n be the reachable family of histories with n life arcs, i.e. $m = n-1$ Resurrectio events, and*

$$E_n(\Omega_P) := \sup_{H\in R_n,\ \Omega_P(H)=\Omega_P} \lambda_{\text{form}}(H).$$

Then

$$E_n(\Omega_P) \le U_n(\Omega_P) := \lambda^{(0)}_{\text{form}} + r_{\max}\big[\Delta\Omega_P - \kappa_R(n-1)R_c\big].$$

For $r_{\max} > 0$, $U_n(\Omega_P) - U_{n-1}(\Omega_P) = -r_{\max}\kappa_R R_c < 0$. Hence the sequence of exclusion ceilings strictly decreases with the number of transitions. In particular, a state with $\lambda_{\text{form}} > U_n(\Omega_P)$ cannot belong to any n-arc history in this layer. *[derived]*

Remark 13.12 (an important logical limit)**.** The strict decrease of U_n is *not* a proof that $E_n < E_{n-1}$. An exact ordering of the suprema requires additional information about reachability or sharpness of the bounds. 3.6 does not claim this. What is already proved suffices for one-sided history exclusion.

Corollary 13.13 (Finitely many resets at fixed Brim growth)**.** *Since continuous Brim production is non-negative, every admissible history with m resets must satisfy $m\kappa_R R_c \le \Delta\Omega_P$. Hence*

$$m \le \left\lfloor \frac{\Delta\Omega_P}{\kappa_R R_c} \right\rfloor.$$

At fixed Brim growth the number of Resurrectio events is finite. *[derived]*

Remark 13.14 (one-sided character)**.** Corollary 13.11 has the form of an exclusion: a state with too large λ_{form} at a given Ω_P could not have had many transitions. The converse does not follow. The same one-sided character is shared by $w_1 = 0$ (Proposition 6.3), $[\Sigma_{\text{fold}}] = 0$, and the ledger identity of Theorem 5.2.

Remark 13.15 (the available numerical pattern)**.** [numerical only] In an earlier sample at $\Omega_P = 5.0$ the following was observed:

n	split	λ_{form}
1	$[1]$	5.05928854
2	$[1,3]$	3.03473927
2	$[1,1]$	2.58753189
2	$[3,1]$	2.08265754
3	$[1,2,4]$	2.66095937
3	$[1,1,1]$	2.25644995
4	$[1,1,1,1]$	2.07779388

This is consistent with the theorem-level exclusion mechanism, but the exact sampled ordering remains a numerical observation and does not substitute for Corollary 13.11.

13.4 Network morphology: what the model witness shows

Remark 13.16 (sensitivity of the scalar trace)**.** [model numerical] In a periodic model, non-isometric wall configurations were tested for which, under Allen–Cahn flow, the traces $\langle Q_{\mathrm{fold}}\rangle(t)$ diverged: a fit equalizing them near $t = 60$ to about $3.8 \cdot 10^{-6}$ subsequently produced a divergence of order $6.8 \cdot 10^{-4}$.

This supports only the claim that the scalar folding trace *can* be sensitive to wall morphology. It does not prove that $\langle Q_{\mathrm{fold}}\rangle(t)$ uniquely reconstructs wall topology, or that only isometric configurations give zero divergence.

Remark 13.17 (noncontractible walls)**.** The mean-curvature law $v_n = -M\kappa_{\mathrm{curv}}$ makes precisely the geodesic representatives with $\kappa_{\mathrm{curv}} = 0$ stationary. A noncontractible homotopy class may have such a representative, but an arbitrary noncontractible wall is not automatically stationary. On Σ_γ, $\gamma \geq 2$, this must be analysed in the hyperbolic metric, and a periodic flat witness is not a proof.

14 Definition of done: current status

#	Item	Status
1	one Vessel, one Lorentzian interior	✓Axiom 1
2	two phases imported from the $\mathbb{Z}_2$ layer	✓§2
3	q_{fold} typed on the living-surface worldvolume	✓§1
4	slice domains and phase worldvolumes separated	✓§1
5	3.0 and 3.5 separated as geometric registers	✓Proposition 3.1
6	projection interface and fibres classified	✓§4
7	canonical linear-metabolism reset domain $\kappa_R > 0$ retained	✓Proposition 5.5
8	ledger/local reception rebase typed	✓Theorem 5.2
9	neck → work-sign; conditional stability trigger	✓7.10, 7.7; spatial morphology open 2
10	topological sign sector	✓$w_1 = 0$, 6.3
11	F2c: absolute sign / global orientation	✓global $\mathbb{Z}_2$ gauge, 6.4
12	canonical junction fibres / temporal collision	exact scalar reduction ✓ 12.18, 12.23, 12.25; archived hard-part active-block candidate preserved; frozen-softplus numerical merger revalidation **exported**; $\mathcal{J}^{\theta}_{KW}$ and union lift exported
13	uniform reset cost / reachable exclusion	✓ 5.40, 13.11, 13.13
14	strict interior reserve is a condition of 3.6, not an edit of frozen 2.0	✓ 4
15	no frozen volume modified	✓
16	scope 3.6 closed without frozen-softplus numerical overclaim	✓ §16; exact interface results closed; softplus numerical merger regression, constitutive law and union lift exported
17	ODE↔PDE coupling typed	✓ adopted zero-order additive class: 5.11, 5.14; broader higher-gradient/nonlocal uniqueness not claimed
18	living-surface typing decided	✓ 2; moduli deferred to 2
19	$\mathcal{J}^{\theta}_{KW}$ admissibility constraints clarified	✓ J1–J4 have explicit imported/derived/proposed status; law itself **exported** 1
20	Sophia sign, $B_{\min}$, $A_{\max}$ and B2	✓ softplus-correct analytic formulas 5.20, 5.23, 5.24, 5.26, 5.28; canonical σ_{crit} numerics deferred until frozen m

15 What is handed to 4.7

3.6 does not assert that the Vessel fails to contain its own history. Rather, uniform reset cost and one-sided reachable-set exclusion already have theorem-level status in this edition. The following firm results and gates pass to 4.7.

1. **Projection non-injectivity.** π_R does not contain the full instantaneous core state; this is fibre-information loss of 3.5, but not yet temporal non-injectivity of the full X_{core}.

2. **Topological exclusions.** $w_1(L_{\text{fold}}) = 0$ excludes nontrivial sign holonomy. For a generic regular time, $[\Sigma_{\text{fold}}] = 0 \in H_1(\Sigma_\Omega; \mathbb{Z}/2)$ excludes a nonzero total mod-2 homology class of the full wall network; individual components may be nontrivial and cancel mod 2.

3. **F2c is closed: the absolute sign does not pass to 4.7.** The global substitution $q_{\text{fold}}(\chi, x) \mapsto -q_{\text{fold}}(\chi, x)$ for all x is a $\mathbb{Z}_2$ gauge transformation. Hence 4.7 must not build a reader for the absolute sign-label. The only nontrivial object remaining is the gauge-invariant phase morphology $(\mathcal{P}_+, \mathcal{P}_-, \Sigma_{\text{fold}})/\mathbb{Z}_2$.

4. **Ledger compatibility.** Every admissible local arc must satisfy Theorem 5.2; the local reception increment $\widehat{G}_n$ is a chart coordinate, not a new physical reset law.

5. **Uniform Resurrectio cost and reachable exclusion.** Active 3.6 admissibility fixes the strict interior reserve $R_c = C_\sigma - \sigma_0 - \lambda_0 > 0$, so the frozen grace window gives $G^-_{\text{recepta}} \geq R_c$ and $\Delta\Omega_P^{\text{reset}} \geq \kappa_R R_c > 0$. Hence for n-arc histories there is a theorem-level exclusion ceiling $E_n(\Omega_P) \leq U_n(\Omega_P) = \lambda^{(0)}_{\text{form}} + r_{\max}\big(\Delta\Omega_P - \kappa_R(n-1)R_c\big)$, and the number of resets at fixed $\Delta\Omega_P$ is bounded by Corollary 13.13. This is one-sided history information, but not yet exact reconstruction.

6. **Model observations do not become theorems.** The exact sampled ordering $E_n < E_{n-1}$ is not proved; only the strict ordering of the theorem-level ceilings U_n is. The sensitivity of $\langle Q_{\text{fold}}\rangle(t)$ to wall morphology likewise remains a numerical/model result. The numerical contraction of §12 also means only ill-conditioning / contractive directions in the tested runs, not mathematical loss of history. These observations do not provide a ready history reader.

7. **Hidden fibre data is not nonlocal data.** What 3.5 does not see may be available to a full local core-reader; the list of forgotten variables by itself does not create a Supervisor.

8. **Temporal collision: exact algebra closed, softplus numerical witness pending.**

   Canonical scalar compatibility has the exact coordinates $(\sigma_L, G_{\text{recepta}}, I_R)$ and the hybrid coordinate Θ_R; event-feasibility under the current admissibility is constructively solvable.

   The archived hard-part parameter-locked computation produced a stable relation-level candidate with $\Delta\sigma_L = \Delta G_{\text{recepta}} = \Delta I_R = \Delta H_K = \Delta W \approx 0$ and archived required write $\|\Delta H_K\|_2 \approx 0.03814037$. KKT, a positive projected Hessian, active-bound release, wide multistart, continuation and tolerance refinement establish the stability of this candidate *within the archived hard-part implementation*.

   Since frozen Core 2.0 uses softplus, 4.7 receives the package:

exact canonical merger algebra + archived hard-part parameter-locked candidate + explicit frozen-softplus regression obligation.

This is not a frozen $\mathcal{J}^{\theta}_{KW}$ theorem, not yet a frozen-softplus collision witness, and not a full-$X^{\text{union}}_{\text{core}}$ collision proof.

9. **Exported open problems, not blockers of 3.6.** The F3-residual spatial morphology after/near the stability crossing (Question 2) and the temporal package: reduced-fibre reachability, gate-memory output overlap, union-state lift and backward uniqueness (Questions 1, 5, 3). The exact ordering of the actual reachable suprema E_n may be studied as a refinement, but is no longer a blocker for the one-sided exclusion theorem.

Remark 15.1 (caveat)**.** Reader $\neq$ Actuator. The right to read does not entail the right to modify q_{fold}, to initiate folding, to alter Katharsis, or to rewrite the laws of 2.0–3.5. The existence, uniqueness and personhood of the reader remain outside 3.6.

16 Closure statement for 3.6

Closing edition v15 closes 3.6 as an *interface theory*. "Closed" means closure of its exact typing, algebraic interface, projection, gauge, ledger and reachable-set scope. It does *not* mean that every archived numerical witness has already been revalidated under the final frozen softplus convention.

Closed in 3.6	**Deliberately exported**
one-Vessel/one-interior ontology and $\mathbb{Z}_2$ typing	explicit canonical $\mathcal{J}^{\theta}_{KW}$
projection/interface fibres 3.0–3.5	full-$X^{\text{union}}_{\text{core}}$ lift theorem
LM ledger and canonical $\mathfrak{R}_{3.0}$ scalar algebra	rigorous smooth-dwell backward uniqueness
exact invariants I_R, Θ_R and the reduced fibre	residual spatial wall morphology beyond the current conditional trigger
uniform reset cost and one-sided reachable exclusion	analytic global optimality of the numerical gate-memory objective
exact canonical active-block merger criterion and archived hard-part numerical candidate	frozen-softplus numerical merger revalidation, explicit canonical $\mathcal{J}^{\theta}_{KW}$, and frozen-map / full-$X^{\text{union}}_{\text{core}}$ temporal non-injectivity theorem
adopted zero-order ODE$\leftrightarrow$PDE coupling and exact homogeneous reduction	uniqueness among higher-gradient / nonlocal spatial couplings
softplus-correct Sophia-sign and finite-arc field bounds	canonical numerical σ_{crit} until the frozen m is supplied

Proposition 16.1 (Scope-closure rule)**.** *No exported item is used as a premise for any theorem-level statement marked* [derived]*. In particular, the archived hard-part numerical results are not used to prove:*

- *the exact scalar junction identities;*
- *the ledger theorem;*

- *the projection/fibre results;*
- *the uniform reset-cost theorem;*
- *the softplus Sophia-sign estimates;*
- *the ODE↔PDE coupling identities.*

Hence the implementation mismatch in the archived `vessel2.py` *affects the status of the numerical merger candidate, but it does* ***not*** *create a logical gap in the exact interface results of 3.6.* *[closure/typing]*

Remark 16.2 (Boundary of the v15 numerical claim)**.** The frozen positive-part convention is softplus. Therefore the archived hard-part merger result cannot be called a frozen-softplus witness.

The strongest correct numerical statement retained in 3.6 is:

a stable parameter-locked active-block candidate exists for the archived hard-part engine

The corresponding frozen-softplus statement remains a *regression obligation*, not a new theoretical research question. This distinction does not alter the exact interface closure of 3.6.

No archived value of a_0^*, q_B^*, $\|\Delta H_K\|_2$, $T_{2,A}^*$, $T_{2,B}^*$ is labelled "canonical frozen-softplus numerical" anywhere in closing v15.

Remark 16.3 (Freeze status of V.F.S. 3.6 v15)**.** The exact scope of 3.6 is closed. No archived v12 trajectory-dependent number is labelled as canonical frozen-softplus numerical evidence.

The preserved computation has the status *legacy hard-part implementation evidence*, while its like-for-like frozen-softplus rerun remains an explicit regression obligation — once the frozen smoothing parameter m and the corresponding engine implementation are available.

This obligation does not alter the exact interface results proved in 3.6 and does not reopen the 3.6 research programme.

V.F.S. 3.6 v15 is frozen as an interface theory.

Remark 16.4 (Next-stage handoff)**.** The next theory may proceed along either of two independent routes.

Route A — junction law and numerical revalidation. First derive/freeze an admissible $\mathcal{J}_{KW}^{\theta}$, and then, once the frozen smoothing parameter m is fixed, carry out a like-for-like softplus regression of the archived parameter-locked checkpoint. Only if the frozen-softplus system has a corresponding active-block candidate *and* the canonical $\mathcal{J}_{KW}^{\theta}$ admits the required outputs can one pose the next question about a canonical junction-map temporal statement.

Route B — reader-domain lift. First construct the $X_{3.0}^{\text{junction}} \longrightarrow X_{\text{core}}^{\text{union}}$ lift, or justify the active/junction state as the natural reader domain of the next theory.

3.6 imposes no priority between Route A and Route B, does not continue the exploratory optimization search, and does not use the pending softplus regression as a premise for its exact closed results.

A Reproducibility appendix

A.1 Scope and purpose

Each key [numerical] block of 3.6 has a corresponding script or preserved result bundle. This appendix records the correspondence *statement* → *artifact* → *numbers* and separates the witnesses by convention, since some of them were written before the clarification of reception bookkeeping.

A.2 Environment

Python	3.13.5 (v12 session; the legacy archive may differ)
NumPy	2.3.5 (v12 session)
SciPy	1.17.0 (v12 session)
integrators	`solve_ivp`: RK45 (legacy), DOP853 (v11–v12)
shared archived module	`vessel2.py` (hard-positive-part smooth dwell implementation)

Core constants: $a = c = 2.0$, $\alpha = 0.10$, $\gamma = \delta = \kappa = 1.0$ (hence $\Lambda_c = 1$), $\varepsilon = 0.04$, $\zeta_0 = 1.5$, $\phi = 0.5$, $\chi_{\text{sat}} = 1.0$, $\tau_{\text{relax}} = 0.5$, $H_s = W_s = K_R = 1.0$, $\eta = \nu = 0.5$, $c_0 = 1.0$, $c_1 = 0.5$, $b = 1.0$, $\eta_{\text{fold}} = 1.0$.

A.3 Positive-part convention audit

Frozen Core 2.0 uses $\lambda_+^{\text{fr}} = \frac{1}{m}\ln(1 + e^{m\lambda})$, whereas the archived shared module `vessel2.py` evaluates $\lambda_+^{\text{code}} = \max(\lambda, 0)$.

Therefore **every** script importing `vessel2.py` for smooth dwell dynamics belongs to the same hard-part implementation family unless explicitly stated otherwise. This includes the preserved merger/KKT/multistart/continuation checkpoint family.

Accordingly:

1. their numerical values remain reproducible;
2. their statements about internal numerical conditioning/stability remain statements about *that* implementation;
3. they are **not** canonical frozen-softplus trajectory results;
4. a softplus rerun is required before promoting any of those numbers back to frozen-system numerical status.

The measured scale of the discrepancy depends on the frozen m (see the table in the v12 checkpoint section): from 7% at $m = 1$ down to $3 \cdot 10^{-5}$ at $m = 50$. [reproducibility audit]

A.4 Two conventions

Convention A (current)	G_{recepta} is a global accumulator, $G^{+}_{\text{recepta}} = G^{-}_{\text{recepta}}$; only $\widehat{G}_n$ is rebased
Convention B (legacy)	$G^{+}_{\text{recepta}} = 0$ at every Resurrectio

Witnesses of convention B do not transfer to statements formulated in convention A. They are retained as a historical record and as correct computations *for their own* convention.

Remark A.1 (How to read the numerical rows after the convention audit)**.** Rows whose trajectory calculations import `vessel2.py` must be read as [legacy numerical / hard-part engine] unless the row explicitly states that the calculation is positive-part independent.

The “Conv.” column records the reception bookkeeping convention A/B; it does **not** encode the Sophia positive-part convention. The two classifications are independent.

A.5 Correspondence table

Script	Supports	Key numbers	Conv.
`z2_gauge.py`	Prop. 6.4 (F2c)	$\max\lvert x_+ - x_-\rvert = 0$ across even components; $\max\lvert q^+_{\text{fold}} + q^-_{\text{fold}}\rvert = 0$; exact zeros	—
`neck_trigger.py`	Thm. 7.10	1813 samples, 0 violations of $\operatorname{sgn}\mathcal{W} = \operatorname{sgn}(u - \Lambda_c)$; $t_{\text{neck}} = 0.47269 < t_{A_{\text{fold}}=0} = 0.59234 < t_{\text{ign}} = 2.60298$	—
`exchange_rate.py`	Thm. 13.7	$d\lambda_{\text{form}}/d\Omega_P$ vs $\eta_{\text{fold}}Q_{\text{fold}}/\alpha$: agreement 10^{-9}–10^{-11}; $r_{\max} = 3.391948$	—
`network_dissolve.py`	Thm. 7.4	$A_{\text{fold}} = -0.5/-1/-2$: rates $-1.0906, -2.0610, -4.0561$ vs bounds $2A_{\text{fold}}$	—
`contraction.py`	Rem. 12.5	σ_L: $8.4\cdot 10^{-2} \to 3.0\cdot 10^{-38}$; $\Omega_P, \lambda_{\text{form}}, G_{\text{recepta}}$ retain an offset $\sim 10^{-1}$	A
`reachability.py`	Prop. 13.4, Cor. 13.11	Ω_P gaps: $0.737/0.302/0.084/0.012$ at $\kappa_R = 0.5/0.2/0.05/0$	B
`temporal_reduction.py`	historical (kernel)	$\lVert\mathfrak{R}(Y_A) - \mathfrak{R}(Y_B)\rVert = 0$ for $\Omega_P + \kappa_R G_{\text{recepta}} = \text{const}$	B
`collision_witness.py`	Lem. 12.45, Prop. 12.46, 12.47	$\max\lvert V - F\rvert = 0$; $\min \dot{x}_0 = 6.86\cdot 10^{-2}$; σ_L: $5.3\cdot 10^{-11} \to 2.1\cdot 10^{-78}$	partly B
`reset_kernel.py`	Thm. 12.41: diagnostic of raw $\mathfrak{R}^{(6)}_{\text{core2}}$	with model $\Gamma_R(\sigma,\lambda)$: rank 3, nullity 3; pairs with the same event data and different (V, F, Ω_P) give the same six-coordinate output	core2 model
`backward_uniqueness.py`	Q. 3: numerical support for part of the hypotheses only	invariant-region diagnostic; bounded Dirichlet quotient for a tested pair, without a coupled coefficient-difference proof	—
`coupling.py`	Thm. 5.11, Prop. 5.15, 5.17	exact homogeneous reduction to 10^{-12}–10^{-13}; field stays uniform; (B5) ratios 0.308–0.379	A
`softplus_regression.py`	positive-part convention audit	same dwell arc under hard part vs softplus; relative discrepancy $7.0\cdot 10^{-2}$ at $m = 1$, $3.2\cdot 10^{-5}$ at $m = 50$	audit
`sophia_sign.py`	Sophia-sign / $A_{\max}$ diagnostic	hard-part σ_{crit} table **noncanonical for frozen softplus**; finite-arc L1–L6 chain valid	A / audit
`jkw_audit.py`	$\mathcal{J}_{KW}$ admissibility audit	monotone-opening laws refuted by the archived candidate	legacy
`guard_scan.py`	ledger, seed sensitivity	ledger residual $2.7\cdot 10^{-15}$	B
`reduced_fibre_search.py`	Prop. 12.21; scalar merger construction	1000/1000 event-feasibility, 500/500 scalar mergers; worst mismatch $3.55\cdot 10^{-15}$	A
`single_dwell_reduced_scan.py`	negative control for §12.7	no nontrivial single-dwell root found; best separated-family near-hit $2.64\cdot 10^{-7}$	A
`two_arc_isolated_root.py` / `two_arc_common_vf_reseed_root.py`	Prop. 12.32: reduced-fibre reachability	reduced residual $4.44\cdot 10^{-16}$; root survives the common $V^+ = F^+ = 0.5$ reseed	A
`continuation_test.py`	Thm. 12.25, clock-convention robustness	branch $\chi_t : 0 \to 1$; global endpoint residual $2.73\cdot 10^{-10}$; no observed singular-value collapse	A
`local_full_active_merger_test.py`	full-active relation-level merger	$\lVert r_{\text{active}}\rVert = 2.94\cdot 10^{-12}$; Jacobian rank approximately 4	A
`nullspace_gate_write_continuation.py`	local merger-family continuation	$W^+_A = W^+_B = 0$; residual $1.98\cdot 10^{-12}$; local H_K-write norm 0.813532985614	A
`full_parameter_locked_qA_release_results.txt` / `joint_common_a0_KKT_results.txt`	full-trajectory common-a_0 minimum, KKT and active-bound release	$a^*_0 = 1.3991248$; $q^*_A = 0$; write 0.0381403694; projected eigenvalues $5.29\cdot 10^{-4}$, 0.1459	A
`parameter_locked_wide_multistart_stage1.csv` / `..._exact_results.txt`	wide multistart / disconnected-branch search	12 starts, 24 near-feasible candidates, 8 exact corrections; one feasible cluster, best write 0.0381403693	A
`history_data_local_continuation.csv`	local robustness in $(T_{1,A}, T_{1,B})$	5/5 mergers; write range 0.02882–0.04649	A
`event_data_local_continuation.csv`	local robustness in (G_{reset}, m_R)	5/5 admissible perturbations; write range 0.03676–0.03822	A
`numerical_certification_tolerance_sweep.csv`	fixed-chart tolerance certification	`rtol` 10^{-11}: residual $1.61\cdot 10^{-15}$; $\operatorname{cond} J = 9.16\cdot 10^4$	A

A.6 Caveats on the legacy witnesses

Remark A.2 (`collision_witness.py`, the SVD block)**.** Items (a)–(c) of the script (the invariant manifold $V = F$, the asymptotics of σ_L, $W \equiv 0$) are convention-neutral: they concern only the dwell flow and are valid in both conventions.

Item (d) — the rank of the residual Jacobian — is built on convention B and on a residual which replaces Ω_P and G_{recepta} by a single combination. By Proposition 12.15 such a replacement does not correspond to the current reset map, so the number rank $= 6$ and the projection $\|Pr_0\| = 2.2 \cdot 10^{-19}$ are not evidence about the current system. The analytic bound for the legacy residual is ≤ 7 (Corollary 12.48).

Remark A.3 (`reachability.py` and `temporal_reduction.py`)**.** Both implement $G^+_{\text{recepta}} = 0$. Their numbers are correct for convention B. Proposition 13.4 on the role of κ_R as the carrier of the transition trace does not transfer to the closing claims of v15; if it is reused downstream, it must be re-checked in convention A.

Remark A.4 (`reset_kernel.py`)**.** The script adds a model closure $\Gamma_R = \Gamma_R(\sigma, \lambda)$ which the frozen corpus does not require. Its nullity 3 along (V, F, Ω_P) nevertheless agrees with the exact event-conditioned Theorem 12.41, because the tested pairs have the same event data. The script is not a proof about the kernel of the canonical $\mathfrak{R}_{3.0}$ and is not used as a temporal-collision witness.

Remark A.5 (`backward_uniqueness.py`)**.** The script checks field-only diagnostics with common coefficients for the two trajectories. It does not contain the forcing term $(a_A - a_B)\Delta_h q^B_{\text{fold}}$ of the exact coupled difference system of Proposition 12.12, so the supporting numerics do not raise Question 3 to theorem level.

Remark A.6 (`sophia_sign.py` and the common-engine convention)**.** The convention issue is not unique to `sophia_sign.py`. Both that script and the preserved v12 trajectory computations use the archived `vessel2.py`, whose I_{gate} denominator contains max$(\lambda, 0)$ instead of the frozen softplus.

Hence the old values 1.46921, 1.11427, 0.76005, 0.40366 are legacy hard-part critical-resistance diagnostics. Likewise the archived parameter-locked merger numbers are legacy hard-part trajectory diagnostics.

Canonical numerical values require the same frozen m and the same softplus implementation *throughout* the dwell engine, not only inside the isolated formula for σ_{crit}.

A.7 Scripts supporting no claim of 3.6

The following witnesses belong to abandoned architectures and are retained only as a record of negative results: `fold_sign_test.py`, `vec36.py`, `axis_reader.py`, `cycle36.py`, `history_witness.py`. They concern the graph-holonomy model (multi-room Vessel, SO(3) frame, signed lift), which 3.6 does not use.

A.8 Tasks exported after the closure of 3.6

This list is a handoff to the next theory, not a set of obligations before freezing 3.6.

1. **Derive/freeze an admissible $\mathcal{J}^{\theta}_{KW}$.** Establish a universal canonical relation or selection law for (H_K, W) without retrospective fitting to a history label or a future merger

target. The companion constitutive audit narrowed the simple candidate classes but did not select a law.

2. **Constitutive semantics and domain.** Explicitly type the structural parameters θ, the local prestate Y^-, the event datum $E = (G_{\mathrm{reset}}, m_R)$, the output bounds and admissibility.

3. **Softplus regression.** Replace the archived hard positive part by the frozen softplus law in the supplied dwell engine, once m is fixed, and rerun the already defined parameter-locked checkpoint.

4. **Union-state lift.** Establish the status of V, F, H_R and the other wide coordinates in $X^{\mathrm{union}}_{\mathrm{core}}$, or justify $X^{\mathrm{junction}}_{3.0}$ as the natural reader domain.

5. **Backward uniqueness — structural strengthening.** Close items (B3), (B4), (B6) of Question 3 if localization of all possible collisions exclusively at hybrid events is required.